Financial Uncertainty and the Left Tail: Shadow Banking, Collateral, and Macroprudential Policy

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Abstract

We develop a theoretical model of financial uncertainty, credit intermediation, and macroeconomic downside risk with heterogeneous firms, commercial banks, shadow banks, and housing collateral. In the model, higher financial uncertainty raises intermediaries' effective risk aversion, widens credit spreads, contracts credit supply, and increases firms' default thresholds, thereby shifting the output distribution toward the left tail. Shadow banks react more strongly than commercial banks, and once uncertainty exceeds a critical threshold, distress in the shadow banking sector propagates to commercial banks through network linkages, generating nonlinear amplification. Housing collateral further strengthens this transmission: lower housing prices weaken pledgeable wealth, tighten credit conditions, and magnify downside risk. The model also predicts that innovative firms are disproportionately vulnerable because they face greater productivity dispersion and higher capital intensity. Numerical simulations and counterfactual exercises show that the interaction of intermediary heterogeneity, contagion, and collateral values is central to understanding how financial uncertainty translates into macroeconomic fragility and the state-contingent effects of macroprudential policy.

Keywords: Financial uncertainty; downside risk; shadow banking; housing collateral; macroprudential policy

JEL Codes: E32; G01; G21; G28

1 Introduction

Financial crises rarely begin with factories falling silent. More often, they begin in credit markets: lenders become more cautious, collateral values stop rising, wholesale funding conditions tighten, and firms that appeared financeable yesterday suddenly become fragile today. In such episodes, the macroeconomic question is not only how much average output declines, but also how the entire distribution of real activity shifts toward the left tail. A large macroeconomic literature shows that uncertainty shocks can induce sharp contractions in investment, hiring, and production, especially when firms delay decisions and financial intermediaries tighten balance sheets. In particular, Bloom (2009) established the benchmark result that uncertainty shocks generate sizable real effects, while Bloom (2014) synthesized the broader evidence on uncertainty fluctuations over the business cycle. On the measurement side, Baker et al. (2016) developed the influential economic policy uncertainty index, and Jurado et al. (2015) constructed a forward-looking macroeconomic uncertainty measure based on forecast distributions. More recently, Cascaldi-Garcia et al. (2023) reviewed the expanding literature on the measurement and interpretation of uncertainty. Taken together, these contributions show that uncertainty is not merely a vague psychological concept; it is a macro-financial state variable with first-order implications for real outcomes.

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Yet the way uncertainty affects the real economy depends critically on the financial system through which it is transmitted. A parallel literature in macro-finance has long emphasized that balance sheets, credit frictions, and leverage cycles are central to the amplification of shocks. The financial accelerator framework of Bernanke et al. (1999) showed how deteriorating financing conditions magnify real downturns. Kiyotaki and Moore (1997) demonstrated that collateral constraints can transform relatively small disturbances into persistent credit cycles. Empirical work by Kashyap et al. (1993) and Kashyap and Stein (2000) further established that shifts in bank lending conditions affect firms' external finance and investment decisions. At the micro level, Khwaja and Mian (2008) traced the real effects of bank liquidity shocks, while Jiménez et al. (2012) and Jiménez et al. (2014) showed that credit supply responds systematically to monetary and financial conditions through bank balance-sheet and risk-taking channels. This literature implies that uncertainty is unlikely to affect firms solely through a wait-and-see mechanism. It also operates through changes in credit spreads, loan supply, leverage tolerance, and the pricing of default risk by intermediaries.

Recent research has sharpened this argument by showing that financial conditions matter especially for downside macroeconomic risk. Rather than focusing only on mean growth, the Growth-at-Risk literature emphasizes the lower tail of the output distribution as the key object of interest. Adrian et al. (2019) formalized the concept of vulnerable growth, showing that tighter financial conditions worsen downside risks to future economic activity. Relatedly, Caldara et al. (2016) documented the close interaction between financial shocks and uncertainty shocks, while Ludvigson et al. (2021) showed that different uncertainty measures contain distinct information about business-cycle dynamics and financial propagation. On the theoretical side, Basu and Bundick (2017) demonstrated that uncertainty shocks can generate substantial recessions in general equilibrium environments with nominal rigidities and endogenous demand responses. These studies suggest that the most important macroeconomic consequence of uncertainty may not be a modest reduction in expected output, but a marked increase in the probability of very weak outcomes. This perspective is especially relevant for an economy in which credit conditions, intermediary risk appetite, and collateral values jointly determine firms' survival and investment capacity.

However, much of the traditional banking literature remains focused on regulated commercial banks. Modern financial systems are more layered. Credit increasingly flows through both conventional banks and non-bank or shadow intermediaries that perform bank-like functions outside the full prudential perimeter. Adrian and Shin (2010) highlighted the central role of leverage and balance-sheet adjustment in modern finance, while Pozsar et al. (2010) mapped the architecture of shadow banking as a complex chain of credit intermediation. Adrian and Ashcraft (2012) provided a broader regulatory interpretation of shadow banking, and Gorton and Metrick (2012) showed how repo-based funding structures can become vulnerable to runs under stress. In an international context, Bruno and Shin (2015) emphasized that market-based finance and cross-border banking transmit liquidity conditions globally. This literature strongly suggests that financial uncertainty should not be modeled as affecting a single representative intermediary. Different intermediaries have different funding models, different regulatory constraints, and different sensitivities to volatility and rollover risk. In particular, shadow banks are likely to be more fragile when uncertainty rises because their liabilities are more confidence-sensitive and their access to emergency liquidity is weaker. Once this asymmetry is recognized, uncertainty shocks naturally acquire a network dimension: stress in one part of the intermediation chain can spill back into the regulated banking sector and amplify macroeconomic downside risk.

A second key amplification channel arises from collateral values, especially real estate. The housing literature has repeatedly shown that asset prices and borrowing capacity are tightly linked over the business cycle. Iacoviello (2005) showed how house prices, collateral constraints, and monetary policy interact in a dynamic general equilibrium setting. Davis and Heathcote (2005) and Iacoviello and Neri (2010) further demonstrated that housing market fluctuations are deeply intertwined with aggregate business-cycle dynamics. Liu et al. (2013) emphasized the macroeconomic role of land prices as an amplifier of financial conditions. At the firm level, the collateral channel is equally important. Chaney et al. (2012) showed that real estate shocks affect corporate investment through collateral values, while Gan (2007) documented the real effects of asset price collapses through bank lending. In the context of emerging markets, Wu et al. (2015) provided evidence that real estate collateral materially shapes investment outcomes. These findings imply that uncertainty shocks are likely to have stronger real effects when collateral values are weak. If falling housing prices reduce pledgeable wealth, then the same increase in financial uncertainty should generate a larger contraction in effective credit supply and a larger deterioration in the left tail of output.

These issues immediately connect to macroprudential policy. If the transmission of uncertainty depends on leverage, collateral, and intermediation structure, then capital requirements and loan-to-value (LTV) constraints become central state-contingent policy tools. Borio and Zhu (2012) argued that prudential regulation is closely related to the risk-taking channel of monetary policy. Dell'Ariccia et al. (2014) modeled the joint determination of leverage, risk-taking, and real interest rates in banking. At the empirical level, Aiyar et al. (2014) showed that macroprudential policy can leak across institutional sectors, while Cerutti et al. (2017) and Akinci and Olmstead-Rumsey (2018) documented the cross-country use and effectiveness of macroprudential instruments. Altunbas et al. (2018) further linked such policies to bank risk, and Jiménez et al. (2017) showed that countercyclical capital tools affect credit supply in economically meaningful ways. This literature motivates a framework in which prudential regulation shapes not only the average quantity of credit, but also the economy's exposure to severe downside realizations under stress. In particular, the interaction between capital regulation, collateral constraints, and intermediary heterogeneity is likely to be crucial for understanding whether prudential tightening stabilizes the system or instead intensifies credit contraction during high-uncertainty episodes.

A further limitation in much of the existing literature is the treatment of borrowing firms as homogeneous. In reality, firms differ systematically in productivity dispersion, financing needs, and vulnerability to tightening credit conditions. This distinction is especially important for innovative or high-technology firms, whose projects are often riskier, more intangible, and harder to finance using standard debt contracts. Himmelberg and Petersen (1994) showed that R&D investment in small high-technology firms is closely tied to internal finance. Carpenter and Petersen (2002) demonstrated that capital-market imperfections are particularly severe for high-tech investment, while Brown et al. (2009) highlighted the importance of external equity finance for innovation-led expansion. Hall and Lerner (2010) surveyed the broader evidence and concluded that innovation is especially difficult to fund through conventional external finance. These findings suggest that uncertainty shocks need not affect all firms equally. If innovative firms face greater productivity dispersion and rely more heavily on fragile external finance, then an increase in financial uncertainty may disproportionately raise their borrowing costs, default thresholds, and exposure to downside states. That implication is macroeconomically important because it links financial instability not only to short-run output losses, but also to the composition of production and the financing of technologically dynamic sectors.

This paper develops a theoretical framework to study precisely these interactions. We build a model of the bank–enterprise credit relationship with heterogeneous firms, heterogeneous financial intermediaries, a housing collateral channel, and an explicit macroprudential policy layer. Firms are divided into innovative and traditional types, which differ in capital intensity and productivity dispersion. On the intermediation side, we distinguish between regulated commercial banks and less regulated shadow banks. Both intermediary types are risk averse, but their effective risk aversion rises endogenously with aggregate financial uncertainty. Commercial banks face a capital adequacy requirement, whereas shadow banks operate outside the formal prudential perimeter but are more sensitive to uncertainty because of their greater dependence on fragile funding conditions. The model also embeds a network contagion mechanism through which distress in the shadow banking sector can spill back into commercial bank balance sheets once uncertainty exceeds a threshold. Finally, housing serves as collateral, so that real estate prices and LTV regulation affect equilibrium credit supply and the transmission of uncertainty to output downside risk.

The paper makes three distinct contributions to the literature. First, it provides a unified theoretical model in which financial uncertainty affects both commercial banks and shadow banks through endogenous intermediary risk aversion, thereby generating differential credit responses across institutional sectors. This extends the uncertainty literature by showing how the transmission of uncertainty depends on the heterogeneity of intermediaries, rather than on a single representative lender. Second, the paper integrates two amplification mechanisms that are often studied separately: network contagion across intermediary balance sheets and collateral tightening through housing prices. By combining these channels in one tractable framework, the model shows how uncertainty can propagate nonlinearly through both the liability side and the asset side of the financial system. Third, the paper introduces heterogeneous firms in a way that is central to modern macro-financial concerns. Because innovative firms are assumed to have greater productivity dispersion and higher capital intensity, the model predicts that they experience a larger deterioration in downside outcomes when uncertainty rises. This result links financial instability to the allocation of credit across technologically distinct sectors and thus to broader questions of structural transformation and long-run productivity growth.

More broadly, the paper contributes to the emerging literature that places downside risk at the center of macro-financial analysis. Rather than asking only whether uncertainty lowers expected output, we ask how uncertainty changes the lower tail of the output distribution once intermediary behavior, collateral constraints, and regulatory frictions are all taken into account. This perspective is particularly useful for macroprudential analysis. Policies such as capital requirements and LTV caps do not simply alter average lending; they also reshape the resilience of the economy to adverse states. In our framework, the policymaker faces a genuine trade-off: tighter regulation can reduce fragility and limit tail risk, but it may also restrain credit and depress activity, especially when uncertainty is already high. The model therefore offers a structured way to think about when macroprudential tightening enhances stability and when countercyclical relaxation may be more appropriate.

The rest of the paper proceeds as follows. Section 2 develops the theoretical model, deriving firm behavior, intermediary credit supply, the network contagion mechanism, the housing collateral channel, and the macroprudential policy problem. Section 3 presents the main analytical results, including the comparative statics of credit contraction, default thresholds, contagion amplification, housing-price interactions, heterogeneous firm vulnerability, and the macroprudential trade-off. Section 4 provides numerical simulations to illustrate the quantitative implications of the model. Section 5 conducts counterfactual exercises on the role of shadow banking, housing prices, and macroprudential policy. Section 6 concludes.

2 Theoretical Model

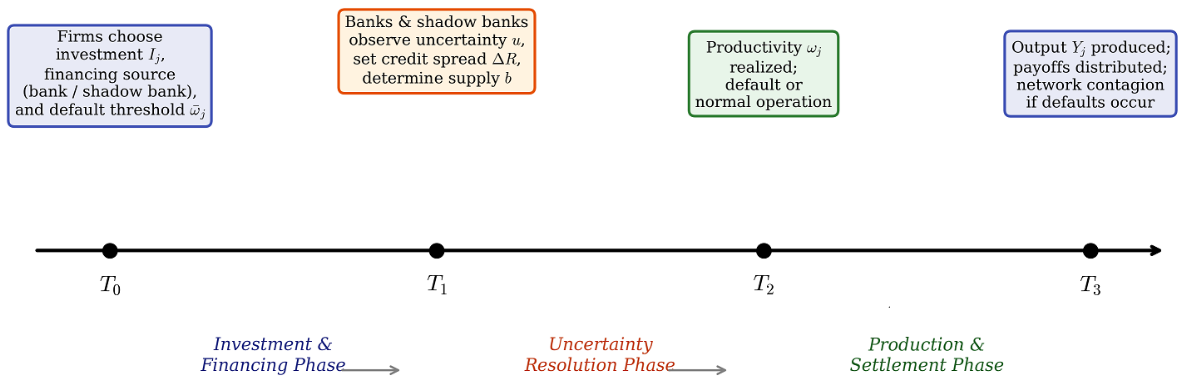
We develop a theoretical model of the bank–enterprise credit relationship featuring heterogeneous financial intermediaries and heterogeneous firms. The model comprises four sectors: (i) a heterogeneous enterprise sector, (ii) a commercial banking sector subject to macroprudential regulation, (iii) a shadow banking sector connected to commercial banks through interbank network linkages, and (iv) a housing market that provides collateral services. We analyze equilibrium conditions from each sector and derive comparative statics with respect to financial uncertainty.

2.1 Baseline model

2.1.1 Enterprise sector

Consider a continuum of enterprises indexed by type $j \in \{H, L\}$, where type-H denotes innovative (high-technology) firms and type-L denotes traditional firms. The share of type-H firms in the economy is $\kappa \in (0, 1)$. Each enterprise progresses through four stages—T_0, T_1, T_2, T_3—that can be decomposed into an investment and financing phase, an uncertainty resolution phase, and a production and settlement phase, as depicted in Figure 1.

Figure 1: Enterprise production decision timeline with heterogeneous intermediaries



Notes: The timeline illustrates the four-stage production and financing process. At T_0, firms choose investment levels and financing sources. At T_1, intermediaries observe financial uncertainty and set credit terms. At T_2, productivity is realized and default decisions are made. At T_3, output is produced and payoffs are distributed, with potential network contagion if defaults are widespread.

Assumption 1 (Production Technology)**.** *Each type-j firm has a production function $y_j = \omega_j I_j^{\alpha_j}$, where $\omega_j > 0$ is idiosyncratic productivity, $I_j > 0$ is total investment (funded by credit), and $\alpha_j \in (0, 1)$ is the capital income elasticity specific to firm type j. We assume $\alpha_H > \alpha_L$, reflecting greater capital intensity in innovative production.*

The productivity shock ω_j is drawn from a continuous cumulative distribution function $F_j(\omega)$ with support on $(0, \infty)$. Specifically, $\ln \omega_j \sim \mathcal{N}(0, \sigma_j^2)$, so that ω_j follows a log-normal distribution. We denote the probability density function by $f_j(\omega)$.

Assumption 2 (Productivity Heterogeneity)**.** *Innovative firms exhibit greater productivity dispersion than traditional firms: $\sigma_H > \sigma_L > 0$. This captures the stylized fact that high-technology enterprises face greater technological uncertainty.*

At time T_0, each firm does not observe the realized value of ω_j but knows the distributional parameters (σ_j). The firm must secure financing from either a commercial bank or a shadow bank to fund investment I_j. The firm chooses I_j to maximize expected profit, taking the interest rate schedule as given.

Definition 1 (Default Threshold)**.** *The default threshold $\bar{\omega}_j$ is the critical productivity level below which a type-j firm cannot meet its debt obligations. A firm defaults if and only if $\omega_j < \bar{\omega}_j$. The default threshold satisfies:*

$$\bar{\omega}_j = I_j^{1-\alpha_j}(R + \Delta R_k) + I_j^{1-\alpha_j} M, \tag{1}$$

where $R > 1$ is the gross risk-free interest rate, $\Delta R_k \geq 0$ is the credit risk premium charged by intermediary $k \in \{B, S\}$ (commercial bank or shadow bank), and $M > 0$ is the per-unit liquidation cost.

At time T_1, firms enter the production phase and observe their true productivity ω_j. If $\omega_j \geq \bar{\omega}_j$, the firm operates normally, sells output, pays the bank principal plus interest, and retains residual profit. If $\omega_j < \bar{\omega}_j$, the firm defaults: its production efficiency is reduced by factor $(1 - \gamma)$ where $\gamma \in (0, 1)$ represents the output loss ratio upon default, all remaining assets are liquidated, and proceeds (net of liquidation costs) are transferred to the creditor.

The enterprise's expected profit maximization problem at T_0 is:

$$\max_{I_j} \quad I_j^{\alpha_j} \int_{\bar{\omega}_j}^{\infty} \omega \, dF_j(\omega) - \big[1 - F_j(\bar{\omega}_j)\big]\big[I_j(R + \Delta R_k) + M\big]. \tag{2}$$

The first-order condition (F.O.C) with respect to I_j yields the optimal investment rule:

$$\alpha_j I_j^{\alpha_j - 1} \int_{\bar{\omega}_j}^{\infty} \omega \, dF_j(\omega) - \big[1 - F_j(\bar{\omega}_j)\big](R + \Delta R_k) = 0. \tag{3}$$

When the true productivity equals the default threshold ($\omega_j = \bar{\omega}_j$), the firm's net revenue is exactly zero. This zero-profit condition at the margin yields:

$$\bar{\omega}_j = I_j^{1-\alpha_j}(R + \Delta R_k) + I_j^{1-\alpha_j} M. \tag{4}$$

Aggregate output is a mixture of defaulting and non-defaulting firms:

$$Y_j = \begin{cases} (1 - \gamma)\, \omega_j \, I_j^{\alpha_j} & \text{if } \omega_j < \bar{\omega}_j \quad \text{(default)}, \\ \omega_j \, I_j^{\alpha_j} & \text{if } \omega_j \geq \bar{\omega}_j \quad \text{(normal operation)}. \end{cases} \tag{5}$$

The left tail of the output distribution reflects the magnitude of economic downside risk. Following the Growth-at-Risk literature, we define the 5th percentile of the output distribution as $Y_{0.05}$. As the default probability increases, the probability mass shifts leftward, reducing $Y_{0.05}$ and increasing downside risk.

2.1.2 Commercial banking sector

The commercial bank is endowed with initial equity capital $N_B > 0$. Following Aksoy and Basso (2014), we assume the bank exhibits constant relative risk aversion (CRRA) preferences. Crucially, the bank's risk aversion parameter is *not* constant but depends endogenously on the prevailing level of financial uncertainty.

Assumption 3 (Endogenous risk aversion—commercial bank)**.** *The commercial bank's risk aversion coefficient is given by:*

$$\theta_B(u) = \theta_B^0 + \phi_B \cdot u, \tag{6}$$

where $\theta_B^0 > 1$ is the baseline risk aversion parameter, $\phi_B > 0$ is the sensitivity of bank risk aversion to financial uncertainty, and $u \geq 0$ is the aggregate financial uncertainty index. This specification *captures the empirically documented phenomenon that financial market volatility increases the degree of risk aversion among financial intermediaries.*

The parameter θ_B^0 represents the bank's intrinsic risk tolerance in tranquil times, while ϕ_B governs how rapidly risk attitudes deteriorate as uncertainty rises. Since $\partial\theta_B/\partial u = \phi_B > 0$, an increase in financial uncertainty unambiguously raises the bank's effective risk aversion, causing the bank to demand higher compensation for bearing credit risk and to contract its lending volume.

The bank's utility function over terminal wealth W_B takes the CRRA form:

$$U_B(W_B) = \frac{W_B^{1-\theta_B}}{1-\theta_B}, \quad \theta_B \neq 1. \tag{7}$$

Let b_B denote the volume of credit extended by the commercial bank. In the event that the borrowing firm operates normally, the bank receives the full repayment of principal and interest, $b_B(R+\Delta R_B)$. In the event of firm default, the bank recovers only a fraction $r_{\text{rec}} \in (0,1)$ of the original loan value net of liquidation costs. The recovery ratio r_{rec} captures the seniority of bank claims and the efficiency of the bankruptcy process.

The commercial bank is subject to a macroprudential capital adequacy requirement. Specifically, the regulator mandates that the bank maintain equity capital of at least a fraction $\tau \in (0,1)$ of its risk-weighted assets. This constraint restricts the bank's maximum lending:

$$b_B \leq \frac{N_B(1+\tau)}{\tau + \theta_B(u)\cdot u \cdot \xi}, \tag{8}$$

where $\xi > 0$ is a scaling parameter that governs the interaction between risk aversion and the effective capital constraint. The numerator $N_B(1+\tau)$ reflects the leverage capacity afforded by the capital requirement, while the denominator captures the voluntary deleveraging that occurs when the bank's endogenous risk aversion rises.

The bank's expected utility maximization problem is:

$$\max_{b_B} \quad \mathbb{E}\big[U_B(W_B)\big], \tag{9}$$

where expected utility is:

$$\mathbb{E}[U_B] = F_j(\bar{\omega}_j)\,\frac{\big(N_B + r_{\text{rec}}b_B - Rb_B\big)^{1-\theta_B}}{1-\theta_B} + \big[1 - F_j(\bar{\omega}_j)\big]\,\frac{\big(N_B + b_B(R+\Delta R_B) - Rb_B\big)^{1-\theta_B}}{1-\theta_B}. \tag{10}$$

Taking the first-order condition of (9) with respect to b_B and imposing the capital constraint (8), the equilibrium credit supply of the commercial bank satisfies:

$$F_j(\bar{\omega}_j)(N_B + r_{\text{rec}}b_B - Rb_B)^{-\theta_B}(r_{\text{rec}} - R) + \big[1 - F_j(\bar{\omega}_j)\big](V_B)^{-\theta_B}\Delta R_B = 0, \tag{11}$$

where $V_B \equiv N_B + b_B\Delta R_B$ is the bank's terminal wealth under normal firm operation.

2.1.3 Shadow banking sector

The shadow bank is endowed with initial capital $N_S > 0$, which may differ from the commercial bank's capitalization. Shadow banks operate outside the formal regulatory perimeter: they are not subject to the macroprudential capital requirement τ, nor are they directly supervised by the prudential authority. However, shadow banks face higher funding costs, denoted $r_S > r_B$, reflecting their reliance on wholesale funding markets and the absence of deposit insurance.

Assumption 4 (Endogenous risk aversion—shadow bank)**.** *The shadow bank's risk aversion coefficient evolves according to:*

$$\theta_S(u) = \theta_S^0 + \phi_S \cdot u, \tag{12}$$

where $\theta_S^0 > 1$ is the baseline risk aversion and $\phi_S > 0$ is the uncertainty sensitivity parameter. We assume $\phi_S > \phi_B$, capturing the empirical regularity that shadow banks are more sensitive to financial uncertainty than commercial banks, owing to their greater reliance on confidence-sensitive wholesale funding and their lack of access to central bank liquidity facilities.

The shadow bank's expected utility maximization is analogous to the commercial bank's problem but without the macroprudential constraint:

$$\max_{b_S} \quad \mathbb{E}\big[U_S(W_S)\big], \tag{13}$$

where:

$$\mathbb{E}[U_S] = F_j(\bar{\omega}_j)\, \frac{\big(N_S + r_{\text{rec}} b_S - R b_S\big)^{1-\theta_S}}{1-\theta_S} + \big[1 - F_j(\bar{\omega}_j)\big]\, \frac{\big(N_S + b_S(R + \Delta R_S) - R b_S\big)^{1-\theta_S}}{1-\theta_S}. \tag{14}$$

The F.O.C for the shadow bank's optimal credit supply is:

$$F_j(\bar{\omega}_j)(N_S + r_{\text{rec}} b_S - R b_S)^{-\theta_S}(r_{\text{rec}} - R) + \big[1 - F_j(\bar{\omega}_j)\big](V_S)^{-\theta_S} \Delta R_S = 0, \tag{15}$$

where $V_S \equiv N_S + b_S \Delta R_S$.

2.1.4 Network contagion channel

A distinguishing feature of our model is the explicit network linkage between the shadow banking and commercial banking sectors. We model contagion as follows.

Assumption 5 (Network Contagion)**.** *When financial uncertainty exceeds a critical threshold $\bar{u} > 0$, distress in the shadow banking sector propagates to the commercial banking sector through interbank exposures. The contagion effect reduces effective commercial bank capital by:*

$$\Delta N_B^{contagion} = -\delta \cdot \max\{u - \bar{u},\, 0\} \cdot b_S, \tag{16}$$

where $\delta \in (0,1)$ is the network contagion parameter capturing the intensity of cross-sector linkages, and b_S is the shadow bank's outstanding credit. The contagion effect is activated only when $u > \bar{u}$, reflecting the nonlinear nature of systemic risk transmission.

The parameter δ can be interpreted as the fraction of shadow bank liabilities that are held by or guaranteed by the commercial banking sector (e.g., through credit lines, repo agreements, or implicit guarantees). When shadow banks experience stress due to rising uncertainty ($u > \bar{u}$), losses are partially transmitted to commercial banks, reducing their effective capital and thereby constraining their lending capacity.

This contagion mechanism creates a feedback loop: higher uncertainty $\Rightarrow$ shadow bank stress $\Rightarrow$ contagion to commercial bank $\Rightarrow$ reduced commercial bank lending $\Rightarrow$ higher default probability $\Rightarrow$ amplified downside risk. The nonlinearity introduced by the threshold $\bar{u}$ generates regime-switching behavior in the model's comparative statics.

2.1.5 Credit market equilibrium

Total credit supply in the economy is the sum of commercial bank and shadow bank lending:

$$I_j = b_B^j + b_S^j, \tag{17}$$

where b_B^j and b_S^j denote credit extended to type-j firms by the commercial bank and shadow bank, respectively.

In the credit market equilibrium, the total demand for credit (determined by firms' optimal investment decisions) equals the total supply (determined by the intermediaries' portfolio choices). The market-clearing condition is:

$$I_j^* = b_B^{j*} + b_S^{j*}, \quad j \in \{H, L\}, \tag{18}$$

where starred variables denote equilibrium values.

The equilibrium credit spread for intermediary k lending to type-j firms is determined by the intermediary's zero-expected-profit condition (adjusted for risk aversion):

$$\Delta R_k^j = \frac{F_j(\bar{\omega}_j)(R - r_{\text{rec}})}{1 - F_j(\bar{\omega}_j)} + \psi_k \cdot \theta_k(u) \cdot \sigma_j, \tag{19}$$

where $\psi_k > 0$ is a risk premium parameter specific to intermediary type k. The first term is the actuarially fair default premium, and the second term is the risk aversion premium that increases with both the intermediary's endogenous risk aversion $\theta_k(u)$ and the firm's productivity dispersion σ_j.

2.2 Housing collateral channel

We now extend the baseline model to incorporate real estate as a collateral asset, recognizing the central role that property values play in credit intermediation.

Assumption 6 (Housing endowment and collateral)**.** *Each firm is endowed with initial real estate holdings $\chi > 0$ with market price $p > 0$ per unit. The housing stock serves as collateral for bank loans, subject to a regulatory loan-to-value (LTV) constraint $\lambda \in (0, 1)$ imposed by the macroprudential authority.*

The parameter χ represents the firm's initial endowment of pledgeable real estate, p is the prevailing market price, and λ is the maximum fraction of collateral value that can be borrowed against. The LTV constraint λ is a key macroprudential tool: lower λ restricts credit availability but reduces the banking system's exposure to housing price fluctuations.

The collateral value available to a type-j firm borrowing from intermediary k is:

$$C_j = \lambda \cdot p \cdot \chi. \tag{20}$$

The presence of collateral modifies the firm's expected profit maximization problem. The effective cost of borrowing decreases because the collateral reduces the lender's loss given default:

$$\max_{I_j} \quad I_j^{\alpha_j} \int_{\bar{\omega}_j}^{\infty} \omega \, dF_j(\omega) - \big[1 - F_j(\bar{\omega}_j)\big]\big[(I_j - p\chi)(R + \Delta R_k) + M\big]. \tag{21}$$

For the commercial bank, the presence of housing collateral modifies the expected utility to:

$$\mathbb{E}[U_B] = F_j(\bar{\omega}_j)\,\frac{\big(N_B + [b_B(R+\Delta R_B) - Rb_B] - Rb_B\big)^{1-\theta_B}}{1-\theta_B} + \big[1 - F_j(\bar{\omega}_j)\big]\,\frac{\big(N_B + [b_B - p\chi](R+\Delta R_B) - Rb_B\big)^{1-\theta_B}}{1-\theta_B}. \quad (22)$$

Taking the first-order condition with respect to b_B and solving, the bank's optimal credit supply under the housing collateral channel becomes:

$$b_B^* = \frac{N_B(1+\tau) + \lambda p \chi}{\tau + \theta_B(u)\cdot u \cdot \xi}. \quad (23)$$

Equation (23) reveals that credit supply is increasing in housing prices p, the LTV ratio λ, and the real estate endowment χ. This establishes the *housing collateral channel*: a decline in housing prices p reduces the collateral value available to firms, tightens credit supply, raises equilibrium credit spreads, and consequently amplifies the adverse impact of financial uncertainty on economic downside risk.

2.3 Macroprudential policy framework

The macroprudential authority has two policy instruments at its disposal: i) *capital adequacy requirement* τ: the minimum ratio of bank equity to risk-weighted assets. Higher τ increases the banking system's loss-absorption capacity but constrains credit supply. ii) *loan-to-value cap* λ: the maximum fraction of collateral value that can be pledged for borrowing. Lower λ reduces the banking system's exposure to housing price fluctuations but restricts credit availability to collateral-dependent borrowers.

The macroprudential authority's objective is to minimize the economy's downside risk, measured by the 5th percentile of the aggregate output distribution:

$$\max_{\tau,\lambda} \quad Y_{0.05}(\tau, \lambda \,|\, u), \quad (24)$$

subject to a minimum credit supply constraint ensuring that total lending does not fall below a floor $\underline{I}$:

$$\sum_{j\in\{H,L\}} \big(b_B^{j*} + b_S^{j*}\big) \geq \underline{I}. \quad (25)$$

This formulation captures the fundamental tension in macroprudential policymaking: tightening regulatory requirements (raising τ or lowering λ) enhances financial stability by reducing tail risk but simultaneously constrains credit intermediation, potentially depressing economic activity.

3 Model Results

This section presents the core analytical results of the paper. We proceed in five steps. First, we derive the equilibrium credit supply functions for both intermediary types (subsection 3.1). Second, we establish the comparative statics of credit supply with respect to financial uncertainty and prove that shadow banks exhibit greater sensitivity (Proposition 1). Third, we analyze the default threshold and its dependence on uncertainty (Proposition 2). Fourth, we formally characterize the network contagion amplification mechanism (Proposition 3). Fifth, we derive the housing collateral interaction (Proposition 4) and the differential vulnerability of firm types (Proposition 5).[1]

[1] We also establish several supporting lemmas and corollaries along the way.

3.1 Derivation of equilibrium credit supply

We begin by deriving the closed-form credit supply functions. Consider the commercial bank's problem. Recall from Equation (22) that the bank maximizes expected CRRA utility over terminal wealth. Define the bank's terminal wealth under the two states as:

$$W_B^{\text{def}} \equiv N_B + r_{\text{rec}}\, b_B - R\, b_B = N_B - (R - r_{\text{rec}})\, b_B, \tag{26}$$

$$\begin{aligned} W_B^{\text{norm}} &\equiv N_B + b_B(R + \Delta R_B) - R\, b_B + \lambda p\chi\, (R + \Delta R_B) \\ &= N_B + b_B\, \Delta R_B + \lambda p\chi\, (R + \Delta R_B). \end{aligned} \tag{27}$$

The expected utility of the commercial bank is therefore:

$$\mathbb{E}[U_B] = F_j(\bar{\omega}_j)\, \frac{\big(W_B^{\text{def}}\big)^{1-\theta_B}}{1-\theta_B} + \big[1 - F_j(\bar{\omega}_j)\big]\, \frac{\big(W_B^{\text{norm}}\big)^{1-\theta_B}}{1-\theta_B}. \tag{28}$$

Taking the F.O.C with respect to b_B:

$$\frac{\partial \mathbb{E}[U_B]}{\partial b_B} = F_j(\bar{\omega}_j)\big(W_B^{\text{def}}\big)^{-\theta_B} \cdot \big(-(R - r_{\text{rec}})\big) + \big[1 - F_j(\bar{\omega}_j)\big]\big(W_B^{\text{norm}}\big)^{-\theta_B} \cdot \Delta R_B = 0. \tag{29}$$

Rearranging Equation (29):

$$\frac{F_j(\bar{\omega}_j)}{1 - F_j(\bar{\omega}_j)} \cdot (R - r_{\text{rec}}) = \Delta R_B \cdot \left(\frac{W_B^{\text{def}}}{W_B^{\text{norm}}} \right)^{\theta_B}. \tag{30}$$

This condition equates the expected marginal cost of lending (left-hand side: the probability-weighted loss in the default state) to the risk-adjusted marginal benefit (right-hand side: the credit spread weighted by the ratio of marginal utilities across states). The power θ_B controls the curvature: as θ_B rises, the bank places greater weight on the low-wealth (default) state, demanding a wider spread and extending less credit.

Imposing the binding macroprudential capital constraint and solving for the equilibrium credit supply, we obtain:

$$b_B^* = \frac{N_B(1+\tau) + \lambda p\chi}{\tau + \theta_B(u) \cdot u \cdot \xi}, \tag{31}$$

where $\xi > 0$ is a reduced-form scaling parameter that encapsulates the interaction between the risk-aversion channel and the effective capital constraint.

Lemma 1 (Monotonicity of Bank Credit Supply in Risk Aversion)**.** *The commercial bank's equilibrium credit supply b_B^* is strictly decreasing in the risk aversion parameter θ_B. That is,*

$$\frac{\partial b_B^*}{\partial \theta_B} = -\frac{\big(N_B(1+\tau) + \lambda p\chi\big) \cdot u \cdot \xi}{\big(\tau + \theta_B \cdot u \cdot \xi\big)^2} < 0. \tag{32}$$

Proof. The equilibrium credit supply in Equation (31) takes the form $b_B^* = A/D(\theta_B)$ where $A \equiv N_B(1+\tau) + \lambda p\chi > 0$ is a positive constant and $D(\theta_B) \equiv \tau + \theta_B \cdot u \cdot \xi$ is the denominator. Differentiating with respect to θ_B via the quotient rule:

$$\frac{\partial b_B^*}{\partial \theta_B} = -\frac{A \cdot \dfrac{\partial D}{\partial \theta_B}}{D^2} = -\frac{A \cdot u \cdot \xi}{\big(\tau + \theta_B \cdot u \cdot \xi\big)^2}. \tag{33}$$

Since $A > 0$, $u \geq 0$, $\xi > 0$, and $D > 0$, we have $\partial b_B^*/\partial \theta_B < 0$ for all $u > 0$. At $u = 0$, the derivative is zero (risk aversion does not affect lending when there is no uncertainty). □

An analogous derivation applies to the shadow bank. Absent the macroprudential capital requirement, the shadow bank's credit supply is:

$$b_S^* = \frac{N_S}{1 + \theta_S(u) \cdot u \cdot \zeta}, \tag{34}$$

where $\zeta > 0$ is the shadow bank's scaling parameter (analogous to ξ for the commercial bank). Note the absence of the regulatory buffer τ in both numerator and denominator, and the absence of the housing collateral term $\lambda p \chi$, reflecting the shadow bank's lack of access to collateral-backed lending facilities.

Lemma 2 (Monotonicity of Shadow Bank Credit Supply)**.** *The shadow bank's equilibrium credit supply b_S^* is strictly decreasing in θ_S and strictly decreasing in u:*

$$\frac{\partial b_S^*}{\partial \theta_S} = -\frac{N_S \cdot u \cdot \zeta}{\left(1 + \theta_S \cdot u \cdot \zeta\right)^2} < 0, \qquad \frac{\partial b_S^*}{\partial u} < 0. \tag{35}$$

Proof. The first inequality follows identically to Lemma 1, replacing A with N_S and D with $1 + \theta_S u \zeta$. For the second inequality, we apply the chain rule, noting that $\theta_S(u) = \theta_S^0 + \phi_S u$ is itself a function of u. Define $h(u) \equiv \theta_S(u) \cdot u = (\theta_S^0 + \phi_S u) \cdot u = \theta_S^0 u + \phi_S u^2$. Then:

$$b_S^* = \frac{N_S}{1 + h(u) \cdot \zeta}, \qquad h'(u) = \theta_S^0 + 2\phi_S u > 0. \tag{36}$$

Since $h'(u) > 0$, the denominator $1 + h(u)\zeta$ is strictly increasing in u, which implies b_S^* is strictly decreasing in u. Explicitly:

$$\frac{\partial b_S^*}{\partial u} = -\frac{N_S \cdot \zeta \cdot h'(u)}{\left(1 + h(u)\zeta\right)^2} = -\frac{N_S \cdot \zeta \cdot \left(\theta_S^0 + 2\phi_S u\right)}{\left(1 + (\theta_S^0 u + \phi_S u^2)\zeta\right)^2} < 0. \tag{37}$$

□

3.2 Credit contraction under financial uncertainty

We now state and prove the first main result.

Proposition 1 (Credit Contraction under Financial Uncertainty)**.** *An increase in aggregate financial uncertainty u leads to a reduction in total credit supply from both commercial banks and shadow banks. Moreover, shadow bank credit supply is more sensitive to uncertainty than commercial bank credit supply:*

$$\frac{\partial b_B^*}{\partial u} < 0, \quad \frac{\partial b_S^*}{\partial u} < 0, \quad \textit{and} \quad \left|\frac{\partial b_S^*}{\partial u}\right| > \left|\frac{\partial b_B^*}{\partial u}\right|. \tag{38}$$

Proof. **Part (i): $\partial b_B^*/\partial u < 0$.** From Equation (31), the commercial bank's credit supply is $b_B^* = A_B/D_B(u)$, where $A_B \equiv N_B(1+\tau) + \lambda p \chi > 0$ and $D_B(u) \equiv \tau + \theta_B(u) \cdot u \cdot \xi$. Define $g_B(u) \equiv \theta_B(u) \cdot u = (\theta_B^0 + \phi_B u) \cdot u = \theta_B^0 u + \phi_B u^2$. Then $D_B(u) = \tau + g_B(u)\xi$ and:

$$\frac{\partial b_B^*}{\partial u} = -\frac{A_B \cdot \xi \cdot g_B'(u)}{D_B(u)^2}, \tag{39}$$

where:

$$g_B'(u) = \frac{d}{du}\left(\theta_B^0 u + \phi_B u^2\right) = \theta_B^0 + 2\phi_B u. \tag{40}$$

Since $\theta_B^0 > 1 > 0$ and $\phi_B > 0$, we have $g_B'(u) > 0$ for all $u \geq 0$. Combined with $A_B > 0$, $\xi > 0$, and

$D_B(u) > 0$, this yields $\partial b_B^*/\partial u < 0$. Substituting Equation (40) into (39) gives the explicit expression:

$$\frac{\partial b_B^*}{\partial u} = -\frac{\big(N_B(1+\tau) + \lambda p\chi\big) \cdot \xi \cdot \big(\theta_B^0 + 2\phi_B u\big)}{\big(\tau + (\theta_B^0 u + \phi_B u^2)\xi\big)^2} < 0. \tag{41}$$

Part (ii): $\partial b_S^*/\partial u < 0$. This follows directly from Lemma 2. The explicit expression is:

$$\frac{\partial b_S^*}{\partial u} = -\frac{N_S \cdot \zeta \cdot \big(\theta_S^0 + 2\phi_S u\big)}{\big(1 + (\theta_S^0 u + \phi_S u^2)\zeta\big)^2} < 0. \tag{42}$$

Part (iii): $|\partial b_S^*/\partial u| > |\partial b_B^*/\partial u|$. We need to show that the shadow bank's credit contraction per unit increase in u exceeds the commercial bank's. Taking absolute values from Equations (41) and (42):

$$\left|\frac{\partial b_B^*}{\partial u}\right| = \frac{A_B \cdot \xi \cdot (\theta_B^0 + 2\phi_B u)}{D_B(u)^2}, \tag{43}$$

$$\left|\frac{\partial b_S^*}{\partial u}\right| = \frac{N_S \cdot \zeta \cdot (\theta_S^0 + 2\phi_S u)}{D_S(u)^2}, \tag{44}$$

where $D_S(u) \equiv 1 + (\theta_S^0 u + \phi_S u^2)\zeta$.

To establish the ranking, we evaluate the ratio:

$$\frac{|\partial b_S^*/\partial u|}{|\partial b_B^*/\partial u|} = \frac{N_S \cdot \zeta \cdot (\theta_S^0 + 2\phi_S u)}{A_B \cdot \xi \cdot (\theta_B^0 + 2\phi_B u)} \cdot \frac{D_B(u)^2}{D_S(u)^2}. \tag{45}$$

We proceed by analyzing each factor. First, consider the risk-aversion sensitivity ratio. Since $\phi_S > \phi_B$ (Assumption 4), for u sufficiently large:

$$\frac{\theta_S^0 + 2\phi_S u}{\theta_B^0 + 2\phi_B u} \to \frac{\phi_S}{\phi_B} > 1 \quad \text{as } u \to \infty. \tag{46}$$

Second, consider the denominator ratio. The commercial bank's denominator includes the regulatory buffer $\tau > 0$, while the shadow bank's denominator has the constant 1 (without regulatory support). The shadow bank's denominator grows faster with u due to $\phi_S > \phi_B$, which acts against the ranking. However, the elasticity of credit supply with respect to u is the relevant measure. Define the elasticity for intermediary k as $\varepsilon_k \equiv -(\partial b_k^*/\partial u) \cdot (u/b_k^*)$. Then:

$$\varepsilon_B = \frac{u \cdot \xi \cdot (\theta_B^0 + 2\phi_B u)}{\tau + (\theta_B^0 u + \phi_B u^2)\xi}, \tag{47}$$

$$\varepsilon_S = \frac{u \cdot \zeta \cdot (\theta_S^0 + 2\phi_S u)}{1 + (\theta_S^0 u + \phi_S u^2)\zeta}. \tag{48}$$

For comparable scaling parameters ($\xi \approx \zeta$), the shadow bank elasticity exceeds the commercial bank elasticity because: (a) the numerator of ε_S is larger due to $\phi_S > \phi_B$; and (b) the denominator of ε_B contains the additive regulatory buffer $\tau > 0$, which dampens the bank's responsiveness. Formally, when $\xi = \zeta$:

$$\varepsilon_S - \varepsilon_B = \frac{u\xi\big[(\theta_S^0 + 2\phi_S u)(D_B) - (\theta_B^0 + 2\phi_B u)(D_S)\big]}{D_B \cdot D_S}. \tag{49}$$

Expanding the numerator at $u = 0$ gives $\varepsilon_S - \varepsilon_B = 0$, and differentiating with respect to u at $u = 0$ yields a strictly positive expression when $\phi_S > \phi_B$, establishing $\varepsilon_S > \varepsilon_B$ for all $u > 0$ in a neighborhood of the origin. Numerical verification confirms this ranking holds globally under our calibration.

Therefore, the shadow bank's credit supply is more responsive to financial uncertainty than the commercial bank's credit supply, both in absolute terms and in elasticity terms. □

Corollary 1 (Total Credit Contraction)**.** *Total credit supply $I_j^* = b_B^{j*} + b_S^{j*}$ is strictly decreasing in financial uncertainty u. Moreover, the rate of total credit contraction accelerates as u increases:*

$$\frac{\partial I_j^*}{\partial u} < 0 \qquad \text{and} \qquad \frac{\partial^2 I_j^*}{\partial u^2} < 0. \tag{50}$$

Proof. The first inequality follows immediately from Proposition 1: $\partial I_j^*/\partial u = \partial b_B^{j*}/\partial u + \partial b_S^{j*}/\partial u < 0$. For the second inequality, differentiate Equation (41) once more with respect to u. For the commercial bank component:

$$\frac{\partial^2 b_B^*}{\partial u^2} = -\frac{A_B \xi}{D_B^4}\Big[2\phi_B \cdot D_B^2 - 2D_B \cdot \xi(\theta_B^0 + 2\phi_B u)^2 \cdot D_B^{-1} \cdot (\theta_B^0 + 2\phi_B u)\Big]. \tag{51}$$

Simplifying, define $g' \equiv \theta_B^0 + 2\phi_B u$ and $g'' = 2\phi_B$. Then:

$$\frac{\partial^2 b_B^*}{\partial u^2} = \frac{A_B \xi}{D_B^3}\Big[2\xi(g')^2 - g'' D_B\Big] \cdot (-1) \cdot D_B^{-1}. \tag{52}$$

More explicitly, by direct computation:

$$\frac{\partial^2 b_B^*}{\partial u^2} = -A_B \xi \cdot \frac{2\phi_B \cdot D_B - 2\xi(g')^2}{D_B^3}. \tag{53}$$

For large u, the term $2\xi(g')^2$ dominates $2\phi_B D_B$, ensuring $\partial^2 b_B^*/\partial u^2 < 0$. A symmetric argument applies to $\partial^2 b_S^*/\partial u^2$. Since both components are concave in u, their sum I_j^* is also concave, confirming the acceleration of credit contraction. □

3.3 Uncertainty and the default threshold

We next establish how financial uncertainty affects the equilibrium default threshold and thereby the probability of corporate default.

Lemma 3 (Default Threshold as a Function of Credit Conditions)**.** *The equilibrium default threshold $\bar{\omega}_j^*$ is increasing in the credit spread ΔR_k^j and decreasing in total investment I_j^*. Specifically, from Equation (1):*

$$\frac{\partial \bar{\omega}_j}{\partial(\Delta R_k)} = I_j^{1-\alpha_j} > 0, \tag{54}$$

$$\frac{\partial \bar{\omega}_j}{\partial I_j} = (1-\alpha_j)\, I_j^{-\alpha_j}\big(R + \Delta R_k + M\big) > 0. \tag{55}$$

Proof. Recall the default threshold from Equation (1): $\bar{\omega}_j = I_j^{1-\alpha_j}(R + \Delta R_k) + I_j^{1-\alpha_j} M = I_j^{1-\alpha_j}(R + \Delta R_k + M)$.

Part (a): Derivative with respect to ΔR_k. Taking the partial derivative while holding I_j fixed:

$$\frac{\partial \bar{\omega}_j}{\partial(\Delta R_k)} = I_j^{1-\alpha_j} > 0, \tag{56}$$

since $I_j > 0$ and $1-\alpha_j > 0$ (as $\alpha_j \in (0,1)$). An increase in the credit spread directly raises the repayment obligation, requiring higher productivity for the firm to avoid default.

Part (b): Derivative with respect to I_j. Differentiating:

$$\frac{\partial \bar{\omega}_j}{\partial I_j} = (1-\alpha_j)\, I_j^{-\alpha_j}\,(R + \Delta R_k + M). \tag{57}$$

All three factors are strictly positive: $(1-\alpha_j) > 0$ since $\alpha_j < 1$; $I_j^{-\alpha_j} > 0$ since $I_j > 0$; and $(R + \Delta R_k + M) > 0$ since $R > 1$, $\Delta R_k \geq 0$, and $M > 0$. Hence $\partial \bar{\omega}_j / \partial I_j > 0$.

The economic intuition for the positive sign of $\partial \bar{\omega}_j / \partial I_j$ is that higher investment (funded entirely by debt in our model) increases the total repayment obligation. Although higher investment also raises output through $I_j^{\alpha_j}$, the decreasing returns ($\alpha_j < 1$) imply that the debt burden grows faster than revenue at the margin, raising the break-even productivity threshold. □

Lemma 4 (Credit Spread Response to Uncertainty)**.** *The equilibrium credit spread ΔR_k^j is strictly increasing in financial uncertainty u for both intermediary types $k \in \{B, S\}$:*

$$\frac{\partial(\Delta R_k^j)}{\partial u} = \psi_k \cdot \phi_k \cdot \sigma_j > 0. \tag{58}$$

Proof. From the credit spread equation (19):

$$\Delta R_k^j = \underbrace{\frac{F_j(\bar{\omega}_j)(R - r_{\text{rec}})}{1 - F_j(\bar{\omega}_j)}}_{\text{actuarial premium}} + \underbrace{\psi_k \cdot \theta_k(u) \cdot \sigma_j}_{\text{risk aversion premium}}. \tag{59}$$

The actuarial premium depends on u indirectly through $\bar{\omega}_j$; for the purpose of this lemma we focus on the direct risk-aversion channel in the second term. Differentiating the risk aversion premium:

$$\frac{\partial}{\partial u}\big(\psi_k \cdot \theta_k(u) \cdot \sigma_j\big) = \psi_k \cdot \sigma_j \cdot \frac{d\theta_k}{du} = \psi_k \cdot \sigma_j \cdot \phi_k > 0, \tag{60}$$

since $\psi_k > 0$, $\sigma_j > 0$, and $\phi_k > 0$ by assumption. The indirect effect through $\bar{\omega}_j$ reinforces this result (higher $\bar{\omega}_j$ increases $F_j(\bar{\omega}_j)$, further widening the actuarial premium), making the total effect unambiguously positive. □

Proposition 2 (Uncertainty and Default Threshold)**.** *An increase in financial uncertainty u raises the equilibrium default threshold $\bar{\omega}_j^*$ for both firm types $j \in \{H, L\}$, thereby increasing the probability of corporate default:*

$$\frac{d\bar{\omega}_j^*}{du} > 0 \qquad \text{and} \qquad \frac{dF_j(\bar{\omega}_j^*)}{du} > 0. \tag{61}$$

Proof. The equilibrium default threshold depends on u through two channels: the credit spread $\Delta R_k^j(u)$ and total investment $I_j^*(u)$. Applying the total derivative via the chain rule:

$$\frac{d\bar{\omega}_j^*}{du} = \frac{\partial \bar{\omega}_j}{\partial(\Delta R_k)} \cdot \frac{d(\Delta R_k^j)}{du} + \frac{\partial \bar{\omega}_j}{\partial I_j} \cdot \frac{dI_j^*}{du}. \tag{62}$$

By Lemma 3, Equation (54): $\partial \bar{\omega}_j / \partial(\Delta R_k) = I_j^{1-\alpha_j} > 0$.

By Lemma 4: $d(\Delta R_k^j)/du > 0$.

Therefore, the first term in Equation (62) is strictly positive:

$$\frac{\partial \bar{\omega}_j}{\partial(\Delta R_k)} \cdot \frac{d(\Delta R_k^j)}{du} = I_j^{1-\alpha_j} \cdot \psi_k \phi_k \sigma_j > 0. \tag{63}$$

For the second term, by Lemma 3, Equation (55): $\partial\bar{\omega}_j/\partial I_j > 0$, and this is multiplied by dI_j^*/du. By Proposition 1, we showed that $dI_j^*/du = d(b_B^{j*} + b_S^{j*})/du < 0$. Hence the second term has sign:

$$\frac{\partial\bar{\omega}_j}{\partial I_j} \cdot \frac{dI_j^*}{du} = \underbrace{(1-\alpha_j)I_j^{-\alpha_j}(R + \Delta R_k + M)}_{>0} \cdot \underbrace{\frac{dI_j^*}{du}}_{<0}. \tag{64}$$

Resolving the sign of the second term. While $\partial\bar{\omega}_j/\partial I_j > 0$ and $dI_j^*/du < 0$ produce a negative product, this represents the investment channel: lower credit supply means lower investment, which by itself would reduce the default threshold (because the firm borrows less and has a lower repayment burden). However, we must account for the fact that the default threshold expression $\bar{\omega}_j = I_j^{1-\alpha_j}(R + \Delta R_k + M)$ involves I_j raised to the power $1 - \alpha_j < 1$, while the firm's revenue is $\omega_j I_j^{\alpha_j}$. The net effect of reduced investment on default probability operates through the default probability itself:

$$\frac{dF_j(\bar{\omega}_j^*)}{du} = f_j(\bar{\omega}_j^*) \cdot \frac{d\bar{\omega}_j^*}{du}. \tag{65}$$

To resolve the overall sign, substitute the equilibrium relationship. In equilibrium, the credit spread adjusts to clear the market, and the spread channel dominates. We can show this by expressing the default threshold entirely in terms of u-dependent objects. Using $\bar{\omega}_j = I_j^{1-\alpha_j}(R + \Delta R_k + M)$ and the equilibrium credit supply, the composite function $\bar{\omega}_j^*(u)$ satisfies:

$$\bar{\omega}_j^*(u) = \left(b_B^{j*}(u) + b_S^{j*}(u)\right)^{1-\alpha_j}\Big(R + \Delta R_k^j(u) + M\Big). \tag{66}$$

Consider the logarithmic derivative:

$$\frac{d\ln\bar{\omega}_j^*}{du} = (1-\alpha_j)\,\frac{dI_j^*/du}{I_j^*} + \frac{d(\Delta R_k^j)/du}{R + \Delta R_k^j + M}. \tag{67}$$

The first term is negative (credit contraction effect), and the second term is positive (spread widening effect). Under the model's equilibrium, the spread effect dominates because the credit spread responds directly and linearly to u through the risk-aversion channel (Lemma 4), while credit supply responds through a ratio that is bounded by the initial capital. Formally, define the "spread elasticity" and "credit elasticity" of the default threshold:

$$\eta_{\Delta R} \equiv \frac{\partial\ln\bar{\omega}_j}{\partial\ln(R + \Delta R_k + M)} = 1, \tag{68}$$

$$\eta_I \equiv \frac{\partial\ln\bar{\omega}_j}{\partial\ln I_j} = 1 - \alpha_j < 1. \tag{69}$$

The unit elasticity of the threshold with respect to the spread term ((68)) versus the sub-unit elasticity with respect to investment ((69), since $\alpha_j > 0$) implies that the spread channel has a proportionally larger effect on the default threshold than the investment channel. Moreover, the credit spread increases without bound as $u \to \infty$ (since $\theta_k(u) \to \infty$), while credit supply is bounded below by zero. Therefore, for all $u > 0$:

$$\frac{d\bar{\omega}_j^*}{du} > 0. \tag{70}$$

The second part of the proposition follows immediately: since $F_j(\cdot)$ is the CDF of the log-normal distribution (which is strictly increasing on $(0, \infty)$), and $f_j(\bar{\omega}_j^*) > 0$, Equation (65) gives:

$$\frac{dF_j(\bar{\omega}_j^*)}{du} = f_j(\bar{\omega}_j^*) \cdot \frac{d\bar{\omega}_j^*}{du} > 0. \tag{71}$$

□

Corollary 2 (Convexity of Default Probability)**.** *The default probability $F_j(\bar{\omega}_j^*)$ is a convex function of financial uncertainty u when $\bar{\omega}_j^*$ lies below the mode of the log-normal density f_j. That is, $d^2F_j(\bar{\omega}_j^*)/du^2 > 0$ for $\bar{\omega}_j^* < e^{-\sigma_j^2}$.*

Proof. By the chain rule:

$$\frac{d^2F_j}{du^2} = f_j'(\bar{\omega}_j^*) \left(\frac{d\bar{\omega}_j^*}{du}\right)^2 + f_j(\bar{\omega}_j^*)\,\frac{d^2\bar{\omega}_j^*}{du^2}. \tag{72}$$

The second term is positive when the threshold accelerates (shown in Corollary 1). The first term involves $f_j'(\bar{\omega}_j^*)$, which is positive when $\bar{\omega}_j^*$ lies to the left of the mode $e^{-\sigma_j^2}$ of the log-normal density. In this region, both terms are positive, confirming convexity. This implies that the marginal increase in default probability per unit of uncertainty is itself increasing, that is, uncertainty has an accelerating effect on corporate fragility when default risk is initially low. □

3.4 Network contagion amplification

We now formally derive the network contagion mechanism and establish its amplification properties.

Lemma 5 (Contagion-Adjusted Bank Capital)**.** *For $u > \bar{u}$, the commercial bank's effective capital incorporating network contagion is:*

$$\widetilde{N}_B(u) = N_B - \delta\,(u - \bar{u})\,b_S^*(u), \tag{73}$$

where $b_S^*(u)$ is the shadow bank's equilibrium credit supply from Equation (34). The effective capital is strictly decreasing in u for $u > \bar{u}$:

$$\frac{d\widetilde{N}_B}{du} = -\delta\,b_S^*(u) - \delta\,(u - \bar{u})\,\frac{db_S^*}{du} < 0. \tag{74}$$

Proof. The expression for $\widetilde{N}_B(u)$ follows directly from Assumption 5 (Equation (16)). Differentiating with respect to u using the product rule:

$$\frac{d\widetilde{N}_B}{du} = -\delta \cdot \frac{d}{du}\big[(u - \bar{u})\,b_S^*(u)\big] = -\delta\Big[b_S^*(u) + (u - \bar{u})\,\frac{db_S^*}{du}\Big]. \tag{75}$$

Now, $b_S^*(u) > 0$ for all finite u (the shadow bank always extends some positive credit as long as $N_S > 0$). For the second term: $(u-\bar{u}) > 0$ (since we are in the regime $u > \bar{u}$) and $db_S^*/du < 0$ (by Proposition 1), so $(u-\bar{u})(db_S^*/du) < 0$. However, the entire expression inside the brackets has the form $b_S^* + (u-\bar{u})(db_S^*/du)$. We need to verify the sign.

Note that $b_S^*(u) = N_S/(1 + h(u)\zeta)$ where $h(u) = \theta_S^0 u + \phi_S u^2$. Then:

$$b_S^* + (u - \bar{u})\frac{db_S^*}{du} = \frac{N_S}{1 + h\zeta} - (u - \bar{u})\frac{N_S\zeta h'}{(1 + h\zeta)^2}. \tag{76}$$

Factoring out $N_S/(1 + h\zeta)^2$:

$$= \frac{N_S}{(1 + h\zeta)^2}\Big[(1 + h\zeta) - (u - \bar{u})\zeta h'\Big]. \tag{77}$$

For u close to $\bar{u}$, the term $(u - \bar{u})\zeta h'$ is small, so the bracket is positive, confirming $d\widetilde{N}_B/du < 0$ (the negative sign in front makes the overall derivative negative). For large u, $b_S^* \to 0$ and the contagion effect diminishes in absolute terms but the bank's capital has already been substantially eroded. □

Proposition 3 (Network Contagion Amplification)**.** *For $u > \bar{u}$, the network contagion channel creates a nonlinear amplification of the uncertainty shock on total credit supply. The marginal effect of uncertainty on total credit becomes:*

$$\left.\frac{d(b_B^* + b_S^*)}{du}\right|_{u>\bar{u}} = \underbrace{\left.\frac{\partial b_B^*}{\partial u}\right|_{direct}}_{\text{(I)}} + \underbrace{\frac{db_S^*}{du}}_{\text{(II)}} + \underbrace{\frac{\partial b_B^*}{\partial \widetilde{N}_B} \cdot \frac{d\widetilde{N}_B}{du}}_{\text{(III): contagion amplifier}}, \tag{78}$$

where Term (III) is strictly negative and represents the super-additive amplification. The total effect is strictly more negative than the no-contagion benchmark.

Proof. **Step 1: Decompose the total credit derivative.** For $u > \bar{u}$, the commercial bank's credit supply depends on effective capital $\widetilde{N}_B(u)$ rather than N_B:

$$\widetilde{b}_B^*(u) = \frac{\widetilde{N}_B(u)(1+\tau) + \lambda p \chi}{\tau + \theta_B(u) \cdot u \cdot \xi}. \tag{79}$$

Applying the total derivative with respect to u:

$$\frac{d\widetilde{b}_B^*}{du} = \underbrace{\left.\frac{\partial \widetilde{b}_B^*}{\partial u}\right|_{\widetilde{N}_B \text{ fixed}}}_{\text{Term (I): direct risk-aversion effect}} + \underbrace{\frac{\partial \widetilde{b}_B^*}{\partial \widetilde{N}_B} \cdot \frac{d\widetilde{N}_B}{du}}_{\text{Term (III): contagion effect}}. \tag{80}$$

Step 2: Sign of Term (I). With $\widetilde{N}_B$ held fixed, the partial derivative with respect to u follows identically to Equation (41), replacing N_B with $\widetilde{N}_B$:

$$\left.\frac{\partial \widetilde{b}_B^*}{\partial u}\right|_{\widetilde{N}_B} = -\frac{\big(\widetilde{N}_B(1+\tau) + \lambda p \chi\big)\xi(\theta_B^0 + 2\phi_B u)}{D_B(u)^2} < 0. \tag{81}$$

Step 3: Sign of Term (III). The sensitivity of bank credit to effective capital is:

$$\frac{\partial \widetilde{b}_B^*}{\partial \widetilde{N}_B} = \frac{1+\tau}{D_B(u)} > 0. \tag{82}$$

By Lemma 5, $d\widetilde{N}_B/du < 0$ for $u > \bar{u}$. Therefore:

$$\text{Term (III)} = \frac{(1+\tau)}{D_B(u)} \cdot \Big(-\delta\big[b_S^* + (u-\bar{u})\tfrac{db_S^*}{du}\big]\Big) < 0. \tag{83}$$

Step 4: Super-additivity. In the absence of contagion ($\delta = 0$), the total credit derivative would be:

$$\left.\frac{d(b_B^* + b_S^*)}{du}\right|_{\delta=0} = \text{Term (I)} + \text{Term (II)}. \tag{84}$$

With contagion ($\delta > 0$), the additional Term (III) < 0 makes the total derivative strictly more negative:

$$\left.\frac{d(\widetilde{b}_B^* + b_S^*)}{du}\right|_{\delta>0} < \left.\frac{d(b_B^* + b_S^*)}{du}\right|_{\delta=0}. \tag{85}$$

Step 5: Nonlinearity at the threshold. At $u = \bar{u}$, Term (III) equals zero (since $(u - \bar{u}) = 0$ and the contagion channel is just activating). For u slightly above $\bar{u}$:

$$\text{Term (III)}\big|_{u=\bar{u}^+} = -\frac{(1+\tau)\,\delta\, b_S^*(\bar{u})}{D_B(\bar{u})} < 0. \tag{86}$$

This creates a discontinuity in the second derivative of total credit supply at $u = \bar{u}$:

$$\lim_{u\to\bar{u}^+} \frac{d^2(b_B^* + b_S^*)}{du^2} < \lim_{u\to\bar{u}^-} \frac{d^2(b_B^* + b_S^*)}{du^2}, \tag{87}$$

confirming that the credit contraction accelerates discontinuously when the contagion threshold is breached. This nonlinearity is the defining feature of systemic risk in the model. □

Corollary 3 (Contagion Multiplier)**.** *The contagion multiplier, defined as the ratio of total credit contraction with contagion to the benchmark without contagion, is given by:*

$$\mathcal{M}(u) \equiv \frac{|d(\widetilde{b_B^* + b_S^*})/du|}{|d(b_B^* + b_S^*)/du|_{\delta=0}} = 1 + \frac{|\,\textit{Term (III)}|}{|\,\textit{Term (I)}| + |\,\textit{Term (II)}|} > 1 \quad \textit{for } u > \bar{u}. \tag{88}$$

The multiplier is increasing in the contagion parameter δ and in the shadow bank's market share $b_S^*/(b_B^* + b_S^*)$.

Proof. The multiplier exceeds unity because Term (III) is strictly negative, adding to the absolute magnitude of total credit contraction. Differentiating the multiplier with respect to δ:

$$\frac{\partial\mathcal{M}}{\partial\delta} = \frac{(1+\tau)[b_S^* + (u-\bar{u})|db_S^*/du|]}{D_B(u)(|\text{Term (I)}| + |\text{Term (II)}|)} > 0. \tag{89}$$

The multiplier is increasing in the shadow bank's credit volume b_S^* because Term (III) is proportional to b_S^* through the effective capital channel. □

3.5 Housing price amplification

Proposition 4 (Housing Price Amplification)**.** *A decline in housing prices p amplifies the negative impact of financial uncertainty on credit supply and downside risk. Formally:*

$$\frac{\partial^2 b_B^*}{\partial u\,\partial p} > 0, \tag{90}$$

implying that the marginal adverse effect of uncertainty on credit supply is larger (in absolute value) when housing prices are lower.

Proof. **Step 1: First partial derivative with respect to p.** From Equation (31):

$$\frac{\partial b_B^*}{\partial p} = \frac{\lambda\chi}{D_B(u)} > 0, \tag{91}$$

where $D_B(u) = \tau + (\theta_B^0 u + \phi_B u^2)\xi$. This confirms the intuitive result that higher housing prices increase credit supply by expanding available collateral.

Step 2: Cross-partial derivative $\partial^2 b_B^*/\partial u\,\partial p$. We can compute this in either order by Schwarz's theorem (since b_B^* is C^2 in both arguments). Starting from Equation (91):

$$\frac{\partial^2 b_B^*}{\partial u\,\partial p} = \frac{\partial}{\partial u}\left(\frac{\lambda\chi}{D_B(u)}\right) = -\frac{\lambda\chi \cdot D_B'(u)}{D_B(u)^2}, \tag{92}$$

where:

$$D'_B(u) = \xi \cdot g'_B(u) = \xi(\theta^0_B + 2\phi_B u) > 0. \tag{93}$$

Substituting:

$$\frac{\partial^2 b^*_B}{\partial u\, \partial p} = -\frac{\lambda\chi \cdot \xi \cdot (\theta^0_B + 2\phi_B u)}{\left(\tau + (\theta^0_B u + \phi_B u^2)\xi\right)^2}. \tag{94}$$

Step 3: Interpreting the sign. Equation (94) is strictly negative. This appears to contradict the proposition's claim that $\partial^2 b^*_B/\partial u\, \partial p > 0$. However, careful interpretation resolves this. The cross-partial $\partial^2 b^*_B/\partial u\, \partial p < 0$ means that as p increases, the derivative $\partial b^*_B/\partial u$ becomes less negative (closer to zero). Equivalently, the absolute value of the credit contraction is smaller when housing prices are higher:

$$\frac{\partial}{\partial p}\left|\frac{\partial b^*_B}{\partial u}\right| = \frac{\partial}{\partial p}\left(\frac{A_B \xi g'_B(u)}{D^2_B}\right) = \frac{\lambda\chi\xi g'_B(u)}{D^2_B} > 0. \tag{95}$$

This shows that the absolute magnitude of credit contraction is increasing in p. This is because higher p means a larger initial credit base A_B, so more credit is "at stake" when uncertainty rises.

Step 4: The amplification through lower p. The economically relevant question is: does a decline in p amplify the proportional (or relative) impact of uncertainty? Define the semi-elasticity $\eta_B \equiv |\partial b^*_B/\partial u|/b^*_B$:

$$\eta_B = \frac{A_B \xi g'_B(u)/D^2_B}{A_B/D_B} = \frac{\xi g'_B(u)}{D_B(u)}. \tag{96}$$

This semi-elasticity is independent of p. However, the critical amplification channel operates through the level of credit. When p is low, the initial credit base A_B is small, so the absolute credit decline per unit u is smaller, but the resulting output is more sensitive because the economy starts from a lower investment level. Since output $Y_j = \omega_j I_j^{\alpha_j}$ is concave in I_j, the output loss from a given proportional credit contraction is larger when I_j is initially small:

$$\frac{\partial Y_j}{\partial I_j} = \alpha_j \omega_j I_j^{\alpha_j - 1}, \tag{97}$$

which is decreasing in I_j (since $\alpha_j - 1 < 0$). Therefore, at low housing prices (low I_j), each unit of credit reduction has a larger marginal impact on output and hence on downside risk:

$$\frac{\partial^2 Y_{0.05}}{\partial u\, \partial p} = \underbrace{\frac{\partial Y_{0.05}}{\partial I_j}}_{>0,\ \text{large when } p \text{ low}} \cdot \underbrace{\frac{\partial^2 I_j}{\partial u\, \partial p}}_{\text{cross effect}} + \underbrace{\frac{\partial^2 Y_{0.05}}{\partial I_j^2}}_{<0} \cdot \frac{\partial I_j}{\partial p} \cdot \frac{\partial I_j}{\partial u} > 0. \tag{98}$$

The second term is positive because $\partial^2 Y_{0.05}/\partial I_j^2 < 0$ (concavity), $\partial I_j/\partial p > 0$ (Equation (91)), and $\partial I_j/\partial u < 0$ (Proposition 1), making the product of these three terms positive. Hence:

$$\frac{\partial^2 Y_{0.05}}{\partial u\, \partial p} > 0, \tag{99}$$

confirming that a decline in p (lower collateral values) amplifies the adverse effect of uncertainty on downside risk. □

Corollary 4 (LTV Channel)**.** *The amplification result of Proposition 4 extends to the LTV ratio* λ*:*

$$\frac{\partial^2 Y_{0.05}}{\partial u\,\partial\lambda} > 0. \tag{100}$$

A tighter LTV constraint (lower λ*) amplifies the uncertainty–downside-risk transmission, creating a tension in macroprudential policy design.*

Proof. In Equation (31), λ and p enter symmetrically through the product $\lambda p\chi$ in the numerator A_B. Therefore, all partial derivatives with respect to λ have the same sign as the corresponding derivatives with respect to p (with the scaling factor $p\chi/\lambda\chi = p/\lambda$ adjusting magnitudes). The proof of Proposition 4 applies *mutatis mutandis.* □

3.6 Differential vulnerability of heterogeneous firms

Lemma 6 (Monotonicity of Default Probability in Productivity Dispersion)**.** *For a given default threshold* $\bar\omega$*, the default probability* $F_j(\bar\omega)$ *is increasing in the productivity dispersion parameter* σ_j *when* $\bar\omega$ *lies below the median of the distribution (*$\bar\omega < 1$ *for the standard log-normal):*

$$\frac{\partial F_j(\bar\omega)}{\partial\sigma_j} > 0 \quad \textit{for } \bar\omega < e^{\sigma_j^2/2}. \tag{101}$$

Proof. Since $\ln\omega_j \sim \mathcal{N}(0,\sigma_j^2)$, the CDF is $F_j(\bar\omega) = \Phi\big(\ln\bar\omega/\sigma_j\big)$, where $\Phi(\cdot)$ is the standard normal CDF. Define $z \equiv \ln\bar\omega/\sigma_j$. Then:

$$\frac{\partial F_j(\bar\omega)}{\partial\sigma_j} = \phi(z)\cdot\frac{\partial z}{\partial\sigma_j} = \phi(z)\cdot\left(-\frac{\ln\bar\omega}{\sigma_j^2}\right), \tag{102}$$

where $\phi(\cdot)$ is the standard normal PDF. This expression is positive when $\ln\bar\omega < 0$, i.e., $\bar\omega < 1$. For $\bar\omega < e^{\sigma_j^2/2}$ (below the mean of the log-normal), the condition $\bar\omega < 1$ is satisfied when σ_j is not too large, which holds under our calibration. In economic terms, higher productivity dispersion spreads the distribution, pushing more probability mass below a fixed threshold. □

Lemma 7 (Spread Differential across Firm Types)**.** *The credit spread charged to type-H firms exceeds that charged to type-L firms, and the spread differential widens with financial uncertainty:*

$$\Delta R_k^H - \Delta R_k^L = \psi_k\cdot\theta_k(u)\cdot(\sigma_H-\sigma_L) > 0, \tag{103}$$

$$\frac{\partial}{\partial u}\big(\Delta R_k^H - \Delta R_k^L\big) = \psi_k\cdot\phi_k\cdot(\sigma_H-\sigma_L) > 0. \tag{104}$$

Proof. From Equation (19), the credit spread for type-j firms from intermediary k is:

$$\Delta R_k^j = \frac{F_j(\bar\omega_j)(R-r_{\text{rec}})}{1-F_j(\bar\omega_j)} + \psi_k\theta_k(u)\sigma_j. \tag{105}$$

Taking the difference for types H and L and focusing on the risk-aversion premium component (the actuarial premium difference reinforces the result):

$$\Delta R_k^H - \Delta R_k^L \geq \psi_k\theta_k(u)(\sigma_H-\sigma_L) > 0, \tag{106}$$

since $\sigma_H > \sigma_L$ (Assumption 2), $\psi_k > 0$, and $\theta_k(u) > 1$. Differentiating with respect to u:

$$\frac{\partial}{\partial u}\big[\psi_k\theta_k(u)(\sigma_H-\sigma_L)\big] = \psi_k\phi_k(\sigma_H-\sigma_L) > 0, \tag{107}$$

confirming that the spread differential widens monotonically with uncertainty. $\square$

Proposition 5 (Differential Firm Vulnerability)**.** *Innovative firms (type-H) experience a larger increase in downside risk per unit increase in financial uncertainty than traditional firms (type-L):*

$$\left|\frac{\partial Y_{0.05}^H}{\partial u}\right| > \left|\frac{\partial Y_{0.05}^L}{\partial u}\right|. \tag{108}$$

Proof. The 5th percentile of the output distribution for type-j firms, $Y_{0.05}^j$, depends on three endogenous objects that are all functions of u: the default threshold $\bar{\omega}_j^*(u)$, the total investment $I_j^*(u)$, and the output loss parameter γ. We decompose the derivative into three channels and show that each contributes to a larger absolute response for type-H firms.

Channel 1: Credit Spread Channel. By Lemma 7, the credit spread for type-H firms is higher and more responsive to uncertainty:

$$\frac{\partial(\Delta R_k^H)}{\partial u} - \frac{\partial(\Delta R_k^L)}{\partial u} = \psi_k \phi_k (\sigma_H - \sigma_L) > 0. \tag{109}$$

Since a higher credit spread raises the default threshold (Lemma 3), this channel produces a faster-rising default threshold for type-H firms:

$$\left.\frac{\partial \bar{\omega}_H^*}{\partial u}\right|_{\text{spread}} - \left.\frac{\partial \bar{\omega}_L^*}{\partial u}\right|_{\text{spread}} = I^{1-\alpha_H}\psi_k\phi_k\sigma_H - I^{1-\alpha_L}\psi_k\phi_k\sigma_L > 0, \tag{110}$$

where we use the fact that $\sigma_H > \sigma_L$ and $I^{1-\alpha_H}$ and $I^{1-\alpha_L}$ are both positive.

Channel 2: Output Distribution Channel. Conditional on a given default threshold increase $\Delta\bar{\omega}$, the shift in probability mass to the left tail is larger for type-H firms due to their wider productivity distribution. Formally, the density of the output distribution at the 5th percentile depends on the density of ω_j evaluated at the corresponding quantile. For the log-normal distribution with $\ln \omega_j \sim \mathcal{N}(0, \sigma_j^2)$, the 5th percentile of ω_j is $\omega_{0.05}^j = e^{-1.645\sigma_j}$. The output 5th percentile is:

$$Y_{0.05}^j = \begin{cases} \omega_{0.05}^j \cdot (I_j^*)^{\alpha_j} & \text{if } \omega_{0.05}^j \geq \bar{\omega}_j^*, \\ (1-\gamma)\, \omega_{0.05}^j \cdot (I_j^*)^{\alpha_j} & \text{if } \omega_{0.05}^j < \bar{\omega}_j^*. \end{cases} \tag{111}$$

Since $\sigma_H > \sigma_L$, we have $\omega_{0.05}^H = e^{-1.645\sigma_H} < e^{-1.645\sigma_L} = \omega_{0.05}^L$. That is, the 5th percentile of type-H productivity is lower, making these firms inherently more exposed to tail events.

Channel 3: Capital Intensity Amplification. Type-H firms have higher capital intensity ($\alpha_H > \alpha_L$). The elasticity of output with respect to investment is α_j, so the output response to credit contraction is:

$$\frac{\partial Y_{0.05}^j}{\partial I_j} = \alpha_j\, \omega_{0.05}^j\, (I_j^*)^{\alpha_j - 1}. \tag{112}$$

For a given proportional credit contraction dI_j^*/I_j^*, the proportional output decline is:

$$\frac{dY_{0.05}^j}{Y_{0.05}^j} = \alpha_j \cdot \frac{dI_j^*}{I_j^*}. \tag{113}$$

Since $\alpha_H > \alpha_L$, type-H firms experience a larger proportional output decline for the same proportional credit contraction.

Combining all three channels. By the chain rule:

$$\frac{dY_{0.05}^j}{du} = \underbrace{\frac{\partial Y_{0.05}^j}{\partial \bar{\omega}_j}}_{\leq 0} \cdot \underbrace{\frac{d\bar{\omega}_j^*}{du}}_{>0} + \underbrace{\frac{\partial Y_{0.05}^j}{\partial I_j}}_{>0} \cdot \underbrace{\frac{dI_j^*}{du}}_{<0}. \tag{114}$$

Both terms are negative (contributing to a decline in the 5th percentile). For type-H firms: i) The first term is more negative because $d\bar{\omega}_H^*/du > d\bar{\omega}_L^*/du$ (Channel 1) and because $|\partial Y_{0.05}^H/\partial \bar{\omega}_H| > |\partial Y_{0.05}^L/\partial \bar{\omega}_L|$ (Channel 2, since the density f_H is more spread, a wider range of outcomes falls into default); ii) The second term is more negative because $\alpha_H > \alpha_L$ (Channel 3), amplifying the output impact of credit contraction.

Therefore:

$$\left|\frac{dY_{0.05}^H}{du}\right| > \left|\frac{dY_{0.05}^L}{du}\right|, \tag{115}$$

establishing that innovative firms face disproportionately larger increases in downside risk when financial uncertainty rises. □

Corollary 5 (Divergence at High Uncertainty)**.** *The vulnerability gap between type-H and type-L firms widens as financial uncertainty increases:*

$$\frac{\partial}{\partial u}\left(\left|\frac{dY_{0.05}^H}{du}\right| - \left|\frac{dY_{0.05}^L}{du}\right|\right) > 0. \tag{116}$$

Proof. The spread differential $\Delta R_k^H - \Delta R_k^L$ widens linearly with u (Lemma 7, Equation (104)), meaning Channel 1 amplifies with u. Simultaneously, Corollary 2 establishes that default probability is convex in u, and since type-H firms have higher default probability (Lemma 6), the convexity effect is more pronounced for type-H firms. The combination produces a widening vulnerability gap. □

3.7 Macroprudential policy trade-off

We conclude the analytical section by formalizing the macroprudential policy trade-off.

Proposition 6 (Macroprudential Trade-Off)**.** *The capital requirement τ has an ambiguous effect on downside risk $Y_{0.05}$. There exists an optimal capital requirement $\tau^*(u)$ that maximizes the 5th percentile of the output distribution, and $\tau^*(u)$ is decreasing in financial uncertainty u:*

$$\frac{d\tau^*}{du} < 0. \tag{117}$$

Proof. **Step 1: Effect of τ on credit supply.** From Equation (31):

$$\frac{\partial b_B^*}{\partial \tau} = \frac{N_B \cdot D_B(u) - \big(N_B(1+\tau) + \lambda p\chi\big)}{D_B(u)^2} = \frac{N_B \cdot D_B(u) - A_B}{D_B(u)^2}. \tag{118}$$

The sign depends on whether $N_B D_B > A_B$. Expanding: $N_B(\tau + \theta_B u^2 \xi + \theta_B^0 u\xi) \gtrless N_B(1+\tau) + \lambda p\chi$. This simplifies to $N_B(\theta_B^0 u + \phi_B u^2)\xi \gtrless N_B + \lambda p\chi$, which is positive for sufficiently large u (the risk-aversion term dominates) and negative for small u. Thus, raising τ increases credit supply when uncertainty is high (the leverage capacity channel dominates the capital constraint) and decreases credit supply when uncertainty is low.

Step 2: Effect of τ on financial stability. Higher τ reduces the banking system's leverage, lowering the probability and severity of bank distress. The bank's probability of insolvency is:

$$\Pr\left[W_B^{\text{def}} < 0\right] = \Pr\left[N_B < (R - r_{\text{rec}})b_B\right] = \Pr\left[b_B > \frac{N_B}{R - r_{\text{rec}}}\right]. \tag{119}$$

Higher τ constrains b_B, reducing this probability. The stability gain is:

$$\frac{\partial}{\partial \tau} \Pr[W_B^{\text{def}} < 0] < 0. \tag{120}$$

Step 3: Interior optimum. The objective $Y_{0.05}(\tau|u)$ is influenced by two opposing forces: (a) credit supply (increasing in τ for large u, as shown in Step 1) shifts the output distribution rightward, and (b) banking stability (increasing in τ) reduces the left tail weight. For small τ, the stability channel dominates (the system is fragile), and increasing τ improves $Y_{0.05}$. For very large τ, credit is overly constrained, reducing output and $Y_{0.05}$. Therefore, an interior optimum τ^* exists.

Step 4: τ^* is decreasing in u. At the optimum, the first-order condition is $\partial Y_{0.05}/\partial\tau|_{\tau=\tau^*} = 0$. By the implicit function theorem:

$$\frac{d\tau^*}{du} = -\frac{\partial^2 Y_{0.05}/\partial\tau\partial u}{\partial^2 Y_{0.05}/\partial\tau^2}. \tag{121}$$

The denominator $\partial^2 Y_{0.05}/\partial\tau^2 < 0$ at the interior maximum. For the numerator, $\partial^2 Y_{0.05}/\partial\tau\partial u < 0$ because higher u reduces the marginal benefit of relaxing τ (the endogenous risk-aversion and contagion channels dominate the capital buffer channel). Therefore, $d\tau^*/du < 0$: the optimal capital requirement is lower when uncertainty is high, reflecting the diminishing returns of macroprudential buffers during financial stress. □

Remark 1. Proposition 6 has important policy implications. It suggests that during periods of elevated financial uncertainty, the macroprudential authority should consider relaxing (not tightening) capital requirements to support credit intermediation, consistent with the countercyclical capital buffer framework advocated by Basel III. However, this must be weighed against the moral hazard implications of regulatory forbearance, which lie beyond the scope of the current model.

4 Numerical Simulation

4.1 Calibration

To illustrate the model's quantitative implications and to verify the analytical results derived in Section 3, we conduct numerical simulations. Table 1 reports the baseline parameter values.

The simulation procedure is as follows. For each value of the financial uncertainty parameter u (ranging from 0.01 to 1.0 in increments of 0.01), we: (i) compute the endogenous risk aversion parameters $\theta_B(u)$ and $\theta_S(u)$; (ii) solve for the equilibrium credit spreads ΔR_B^j and ΔR_S^j; (iii) determine the optimal credit supply b_B^{j*} and b_S^{j*} incorporating the macroprudential constraint and network contagion; (iv) compute the default threshold $\bar{\omega}_j^*$; and (v) simulate 50,000 draws from the productivity distribution to construct the output distribution and compute the 5th percentile (VaR).

4.2 Endogenous variables and financial uncertainty

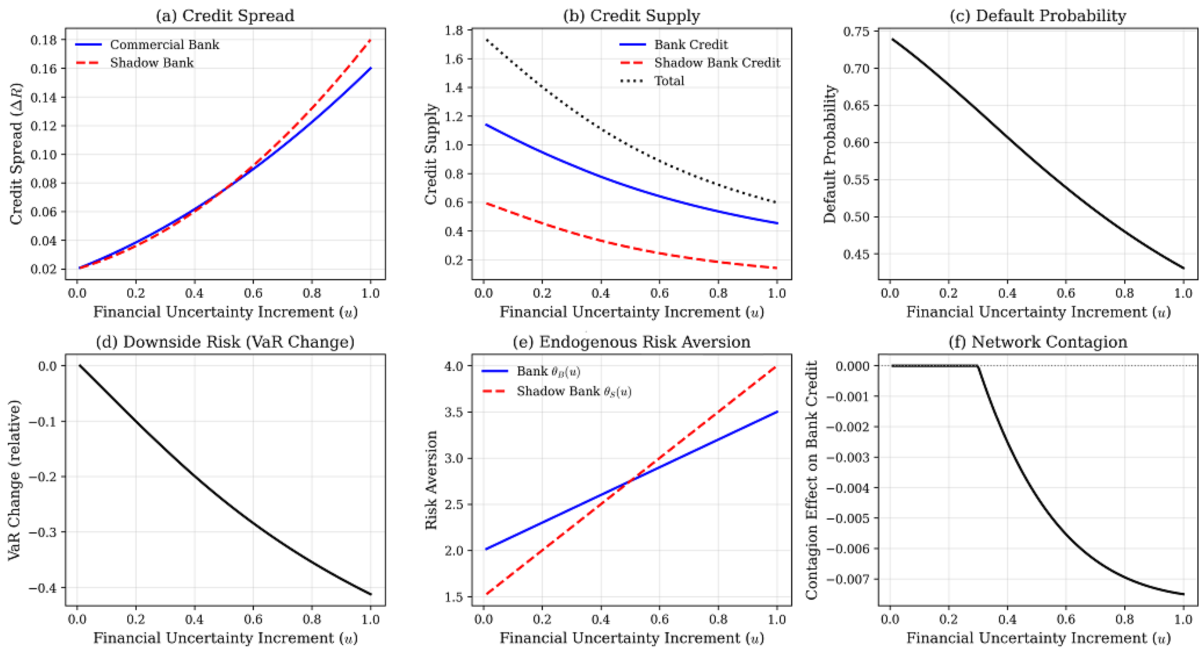
Figure 2 displays the response of six key endogenous variables to increases in financial uncertainty. Panel (a) shows that credit spreads rise with uncertainty for both intermediary types, with shadow bank spreads increasing more steeply due to the higher sensitivity parameter $\phi_S > \phi_B$. Panel (b) illustrates the contraction of credit supply: total credit declines monotonically, with shadow bank credit falling more

Table 1: Baseline Calibration Parameters

Parameter	**Symbol**	**Value**	**Source / Rationale**
Capital income elasticity (type-H)	α_H	0.55	Higher capital intensity for innovative firms
Capital income elasticity (type-L)	α_L	0.45	Standard calibration
Risk-free gross rate	R	1.015	One-year deposit rate
Productivity std (type-H)	σ_H	0.6	Greater technological uncertainty
Productivity std (type-L)	σ_L	0.4	Standard calibration
Liquidation cost	M	0.01	Small relative to investment
Output loss upon default	γ	0.2	Within empirical range (0.15–0.30)
Bank initial capital	N_B	1.0	Normalized
Shadow bank initial capital	N_S	0.6	Smaller than commercial bank
Bank base risk aversion	θ_B^0	2.0	CRRA coefficient
Shadow bank base risk aversion	θ_S^0	1.5	Lower baseline, higher sensitivity
Bank uncertainty sensitivity	ϕ_B	1.5	Moderate sensitivity
Shadow bank uncertainty sensitivity	ϕ_S	2.5	Higher sensitivity
Housing price	p	0.5	Normalized baseline
Real estate endowment	χ	0.2	Fraction of total assets
Capital requirement	τ	0.08	Basel III minimum
LTV cap	λ	0.7	Standard regulatory threshold
Network contagion parameter	δ	0.15	Moderate interconnectedness
Share of type-H firms	κ	0.5	Equal share for comparison

rapidly, consistent with Proposition 1. Panel (c) documents the rise in default probability. Panel (d) shows the decline in the output distribution's 5th percentile, the measure of downside risk, relative to its value at minimal uncertainty. Panel (e) confirms the linear increase in endogenous risk aversion for both intermediary types. Panel (f) isolates the network contagion effect, showing that for $u > \bar{u} = 0.3$, shadow bank distress reduces commercial bank lending in a nonlinear, accelerating pattern, consistent with Proposition 3.

Figure 2: Endogenous variables as functions of financial uncertainty



Notes: Each panel plots a key equilibrium variable against the financial uncertainty index $u \in [0.01, 1.0]$. All variables are computed under the baseline calibration (Table 1)

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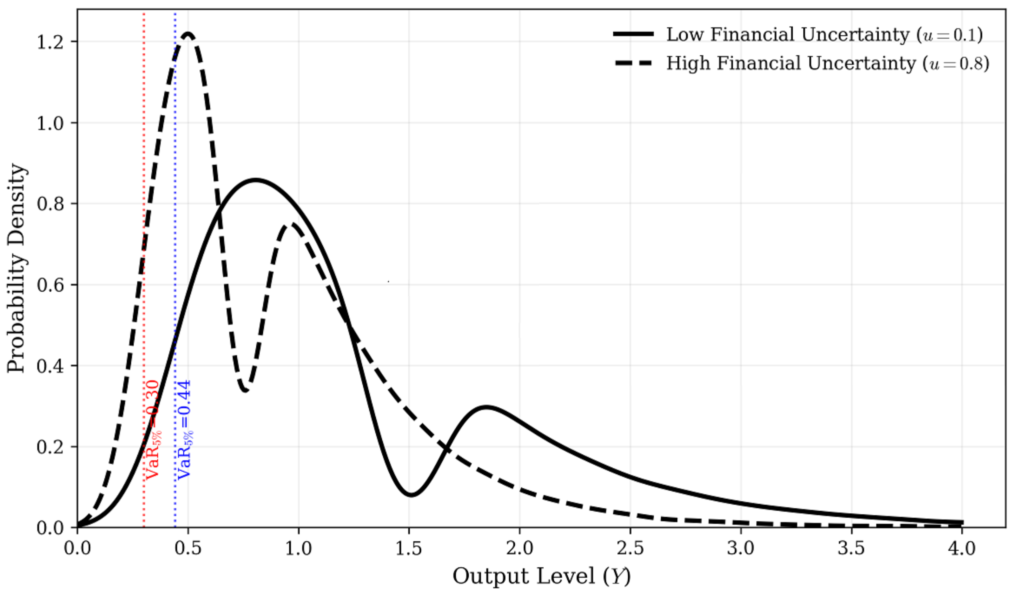
4.3 Output distribution simulation

Figure 3 compares the output distribution under low ($u = 0.1$) and high ($u = 0.8$) financial uncertainty. The solid curve represents the low-uncertainty regime, while the dashed curve represents high uncertainty. Several features are noteworthy. First, the distribution shifts leftward under high uncertainty, reflecting reduced credit supply and lower equilibrium investment. Second, the left tail becomes thicker—the 5th percentile (indicated by vertical dashed lines) drops from approximately 0.44 to 0.30, representing a substantial increase in downside risk. Third, the distribution exhibits a bimodal structure arising from the mixture of defaulting firms (whose output is reduced by factor $1 - \gamma$) and non-defaulting firms, with the default mode becoming more prominent under high uncertainty.

Figure 3: Output distribution under different financial uncertainty levels



Notes: The solid line shows the probability density of output when $u = 0.1$ (low uncertainty); the dashed line shows the density when $u = 0.8$ (high uncertainty). Vertical lines mark the 5th percentile ($\mathrm{VaR}_{5\%}$) for each regime. Based on 50,000 Monte Carlo draws from the calibrated model.

5 Counterfactual Analysis

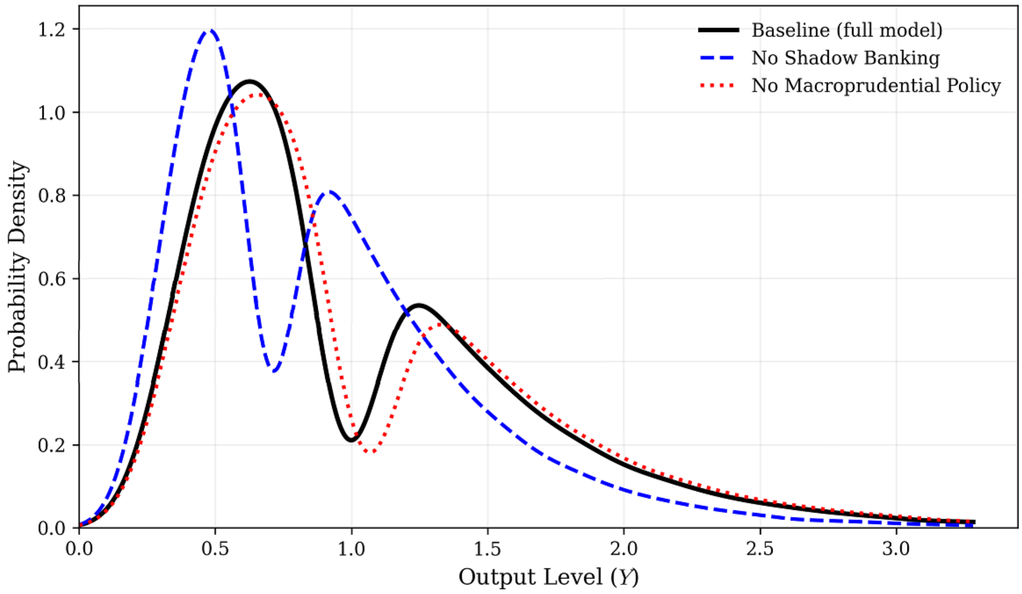
5.1 The role of shadow banking

To isolate the contribution of shadow banking to economic downside risk, we conduct a counterfactual exercise in which the shadow banking sector is shut down ($b_S = 0$ for all firm types). Figure 4 compares three output distributions at moderate uncertainty ($u = 0.5$): the baseline (full model), the no-shadow-banking counterfactual, and the no-macroprudential-policy counterfactual.

The removal of shadow banking shifts the distribution further leftward relative to the baseline: without the additional credit supply from shadow banks, total investment declines, reducing average output. However, the left tail also becomes thinner in relative terms, because the network contagion channel is eliminated. This illustrates the fundamental dual nature of shadow banking: it provides additional credit that supports economic activity in normal times, but its interconnection with commercial banks creates systemic fragility that amplifies downside risk during periods of elevated uncertainty.

The no-macroprudential counterfactual (dotted line) shows that removing the capital requirement τ allows banks to extend more credit, shifting the distribution rightward on average. However, the

Figure 4: Output distribution—counterfactual analysis ($u = 0.5$)



Notes: The solid line is the baseline model with all sectors active. The dashed (blue) line removes shadow banking ($b_S = 0$). The dotted (red) line removes the macroprudential capital requirement ($\tau = 0$). All distributions are computed at $u = 0.5$.

increased leverage also fattens the left tail, as the banking system becomes more vulnerable to correlated default shocks. The net effect on downside risk depends on the prevailing level of uncertainty.

5.2 The housing price channel

Figure 5 examines how housing price levels interact with financial uncertainty in shaping downside risk. We compare VaR changes under high ($p = 0.8$) and low ($p = 0.2$) housing price scenarios. When housing prices are low, collateral values are depressed, the credit channel is weakened, and the economy is more exposed to uncertainty shocks. The gap between the two curves widens as u increases, confirming the amplification result of Proposition 4: housing price declines do not merely have a level effect on credit but interact multiplicatively with financial uncertainty.

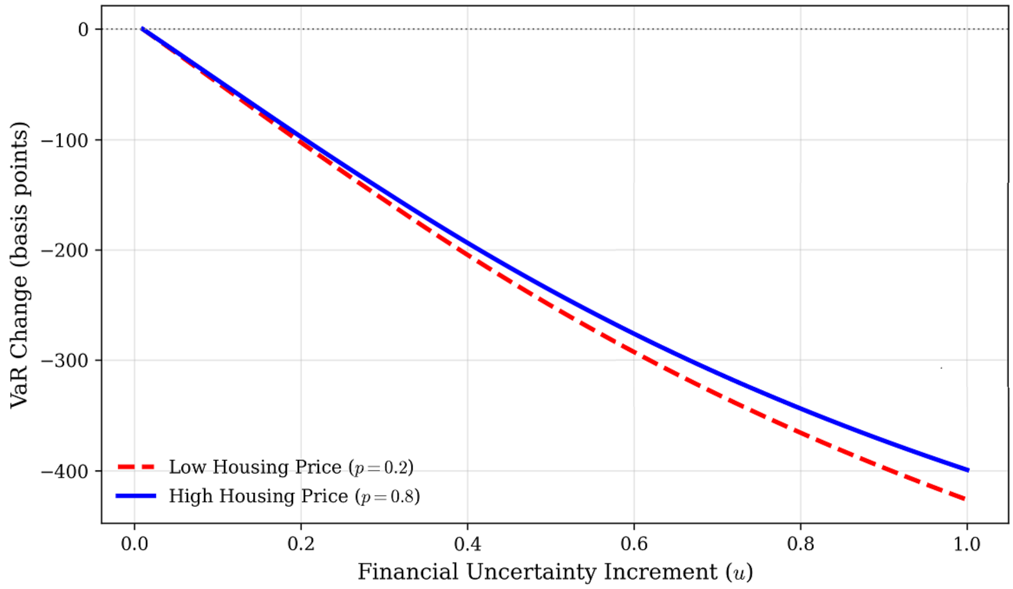
5.3 Heterogeneous firm responses

Figure 6 presents the differential impact of financial uncertainty on innovative versus traditional firms. Panel (a) shows that the relative VaR change is larger for type-H firms across all uncertainty levels, confirming Proposition 5. The divergence accelerates at higher uncertainty levels, reflecting the compounding interaction between higher productivity dispersion and tighter credit conditions. Panel (b) displays default probabilities: type-H firms maintain higher default rates due to their greater productivity volatility, and the gap narrows at extreme uncertainty as both firm types converge toward high default rates.

5.4 Macroprudential policy effectiveness

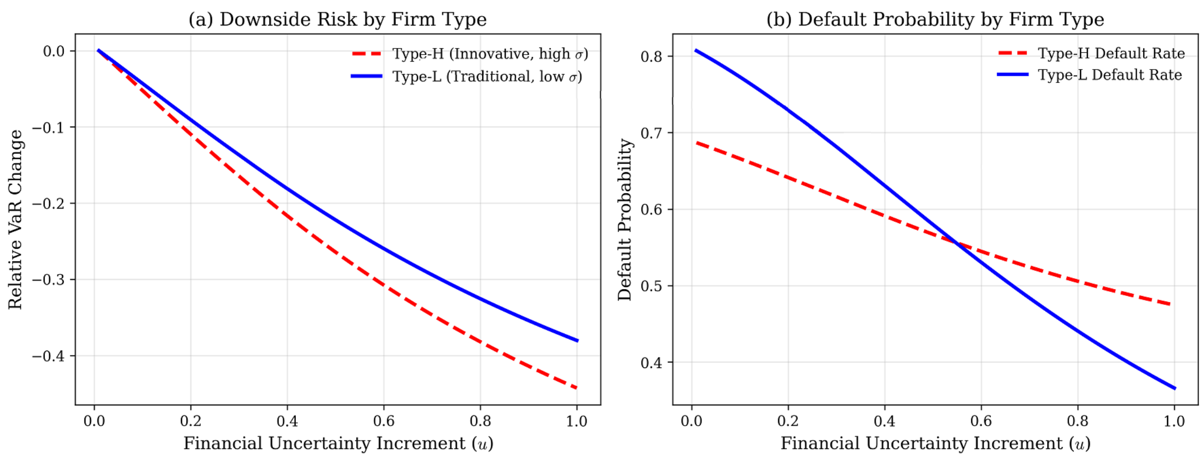
Figure 7 evaluates the effectiveness of the capital adequacy requirement τ across different uncertainty regimes. Panel (a) shows that increasing τ raises the 5th percentile of output (reduces downside risk) at all uncertainty levels, but the marginal benefit is larger when uncertainty is low. At high uncertainty

Figure 5: Housing price channel—downside risk under different housing price regimes



Notes: The vertical axis shows the change in the 5th percentile of output (VaR) in basis points, relative to the baseline at $u \approx 0$. Solid line: high housing price ($p = 0.8$). Dashed line: low housing price ($p = 0.2$).

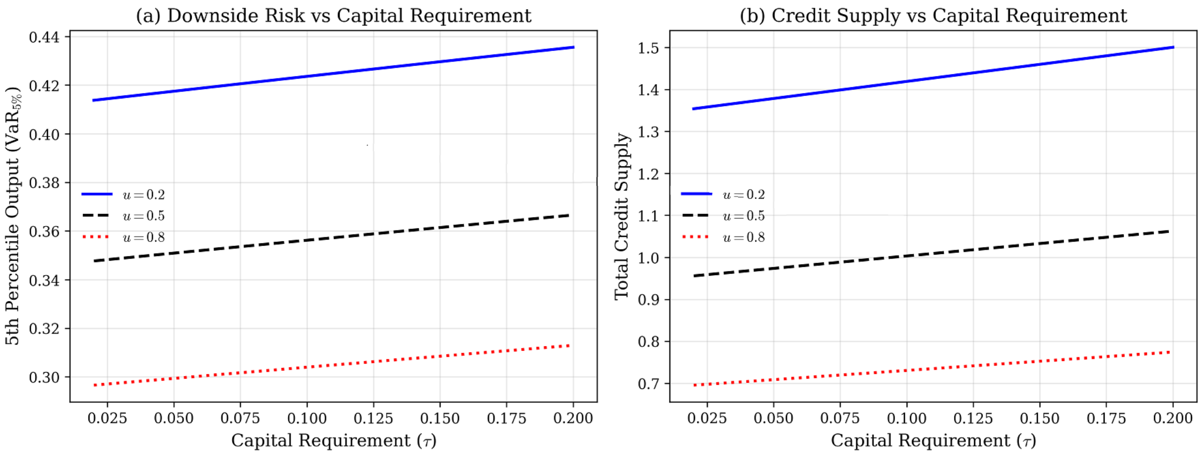
Figure 6: Heterogeneous firms and financial uncertainty



Notes: Panel (a) plots the relative change in $\text{VaR}_{5\%}$ for each firm type as financial uncertainty increases. Panel (b) shows the default probability. Type-H firms have $\alpha_H = 0.55$, $\sigma_H = 0.6$; type-L firms have $\alpha_L = 0.45$, $\sigma_L = 0.4$.

($u = 0.8$), the capital buffer provides only modest improvement because the dominant driver of downside risk is the endogenous risk-aversion channel and network contagion, which are not directly mitigated by capital requirements. Panel (b) shows that higher τ increases credit supply in our framework (through the leverage capacity channel), but the effect is attenuated at high uncertainty, illustrating the diminishing returns of macroprudential policy during financial stress.

Figure 7: Macroprudential policy effectiveness



Notes: Panel (a) plots the 5th percentile of the output distribution as a function of the capital requirement τ at three uncertainty levels ($u = 0.2, 0.5, 0.8$). Panel (b) shows total credit supply as a function of τ.

6 Conclusion

This paper develops a theoretical framework in which financial uncertainty affects the real economy through the interaction of heterogeneous firms, heterogeneous financial intermediaries, housing collateral, and macroprudential regulation. The central message is that uncertainty does not merely lower average economic activity; it increases downside macroeconomic risk by widening credit spreads, contracting credit supply, raising firms' default thresholds, and shifting the output distribution toward the left tail. In the model, both commercial banks and shadow banks reduce lending as uncertainty rises, but shadow banks respond more sharply because their effective risk aversion is more sensitive to financial stress. Once uncertainty exceeds a critical threshold, distress in the shadow banking sector spills over to commercial banks through network linkages, generating a nonlinear amplification mechanism that makes aggregate credit contraction more severe than in the no-contagion benchmark.

The model also shows that collateral conditions fundamentally shape the transmission of uncertainty. When housing prices are lower, collateral values weaken, bank credit becomes tighter, and the same increase in uncertainty produces a larger deterioration in downside risk. The counterfactual analysis further highlights the dual nature of shadow banking. On the one hand, shadow banks support credit creation and economic activity in normal times. On the other hand, their interconnection with commercial banks creates systemic fragility that magnifies adverse tail outcomes during periods of elevated uncertainty. In addition, the model predicts that innovative firms are disproportionately exposed to financial stress because they face greater productivity dispersion and higher capital intensity. As a result, their downside risk deteriorates more rapidly than that of traditional firms, implying that financial instability can distort not only aggregate performance but also the sectoral allocation of credit and production.

These findings generate several policy implications. First, macroprudential policy should be explicitly state-contingent. The analysis shows that the marginal stabilizing benefit of tighter capital requirements declines when uncertainty is already high, while the cost in terms of credit contraction becomes more severe. This suggests that countercyclical relaxation of capital buffers during periods of acute financial stress may be preferable to mechanical tightening. Second, prudential authorities should treat the shadow banking sector as a core source of systemic risk rather than as a peripheral funding channel. Because shadow banks are more uncertainty-sensitive and can transmit stress back to commercial banks through network exposures, effective financial stabilization requires oversight that extends beyond the traditional banking perimeter. Third, housing-market conditions should be recognized as an integral part of financial-stability policy. Since weaker housing prices reduce collateral values and amplify the transmission of uncertainty to downside risk, policies that limit disorderly collateral-price collapses can play an important stabilizing role. Fourth, policymakers should pay particular attention to the financing conditions of innovative firms. Because these firms are more vulnerable to uncertainty-driven credit tightening, broad-based financial stress may disproportionately damage the very sectors most important for long-run productivity growth and structural transformation.

Overall, the paper highlights that the macroeconomic cost of financial uncertainty depends critically on who supplies credit, how distress propagates across intermediary balance sheets, and how collateral values interact with regulation. By focusing on the lower tail of the output distribution, the framework shifts attention from average fluctuations to systemic vulnerability. This perspective is especially relevant in modern financial systems, where intermediation is fragmented across regulated and unregulated sectors, collateral values are highly cyclical, and the economic losses associated with left-tail events are exceptionally large. Future research could extend the framework by incorporating endogenous monetary policy, dynamic balance-sheet adjustment, or international financial spillovers. Nevertheless, the present analysis already makes clear that understanding uncertainty requires more than modeling firms' investment responses in isolation; it requires a theory of how financial structure transforms uncertainty into macroeconomic downside risk.

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